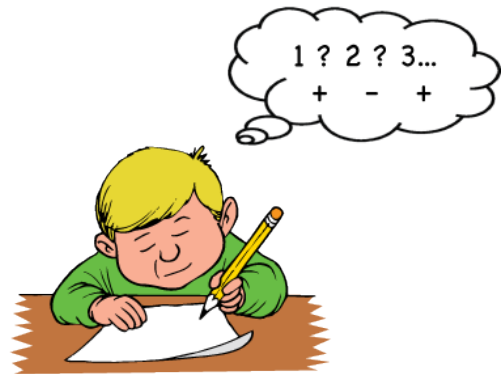
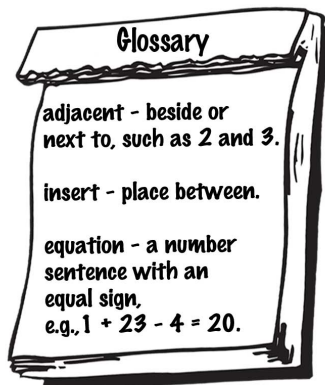


Problem

- a) Tommy has a problem. He knows the numbers below form an equation if he inserts addition and subtraction signs, and uses two adjacent digits together, in order, to form a two-digit number. Help Tommy make the equation true by inserting addition and subtraction signs, and deciding which two adjacent digits to combine.

$$1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 = 100$$

- b) Solve the problem with the digits in reverse order: $9 \ 8 \ 7 \ 6 \ 5 \ 4 \ 3 \ 2 \ 1 = 100$.



Extension :

1. Solve the problem with the digits in the same order as in part a), but this time use any of the operations addition, subtraction, multiplication, division, powers, and square roots. You may also use two, or three, adjacent digits to form new digits.

Hints**Part 1a)**

Hint 1 - How does the problem change once you've combined two digits? For example, if you combine 2 and 3 to form 23, what must be the sum of all the other numbers?

Hint 2 - What combinations of two digits are impossible? Why?

Part 1b)

Hint 1 - What combinations of two digits reversed will not work? Why?

Solution

- a) The possible two-digit combinations are 12, 23, 34, 45, 56, 67, 78, and 89. The first five are too small (e.g., for 56, the sum of the remaining digits is $1 + 2 + 3 + 4 + 7 + 8 + 9 = 34$, and $56 + 34 = 90$), so only 78 and 89 are possible. For 78, two solutions are:

$$1 + 2 + 3 - 4 + 5 + 6 + 78 + 9 = 100, \text{ or } -1 + 2 - 3 + 4 + 5 + 6 + 78 + 9 = 100$$

(there may be others).

The larger combination, 89, leaves a remainder of 11, which must be constructed from sums and/or differences of 1, 2, 3, 4, 5, 6, 7. Since there are 4 odd numbers and 3 even numbers, only an even sum or difference can occur, because sum and differences of two odd numbers are always even. Thus 11 cannot be constructed so 89 is not possible from these numbers.

- b) With the digits reversed, the same conditions hold as in a). The combinations 54 and 32 are too small, but there are two combinations of adjacent digits that work:

$$9 + 8 + 76 + 5 - 4 + 3 + 2 + 1 = 100, \text{ and } 98 + 7 - 6 + 5 - 4 + 3 - 2 - 1 = 100$$

Extension : Here are several solutions.

$$1 \times 2 \times 3 - 4 + 5 + 6 + 78 + 9 = 100$$

$$1 \times 2 - 3 + 4 - 5 + 6 + 7 + 89 = 100$$

$$123 - 4 + 5 - 6 - 7 - 8 - \sqrt{9} = 100$$

$$1 + 2^3 - 4 + 5 - 6 + 7 + 89 = 100$$

$$\frac{1}{2} \times 34 + (5 \times 6) + (7 \times 8) - \sqrt{9} = 100$$