

Emmy Noether - Circle 3 for 2010-2011



Part I: Problems

Problem 1

Harry and Sam, and their sons Micah and Todd, wish to cross a river. Their small boat can carry only the mass of one man, or of one or two boys. What is the minimum number of one-way trips the boat must make across the river in either direction in order to get all four people to the opposite shore?

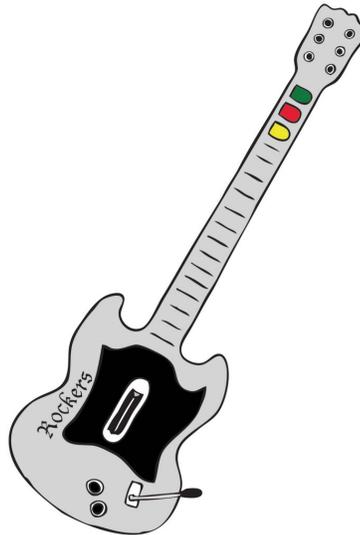


Problem 2

Bella loves to play "Rockers" guitar. Having devoted 1500 hours to this game, she has some questions about the available combinations of notes. For example, if there were just two colours (notes), say Red and Yellow (labelled R and Y for convenience), then the possible combinations (i.e., two-note riffs), would be RR, RY, YR, and YY.

When playing "Rockers" on Easy Level, there are three colours to press, Green, Red, and Yellow (label them G, R, and Y), so there are 3 one-note riffs.

- a) If you play a two-note riff on Easy Level, what are all the possibilities you could play for this riff? Enter the total number of riffs in the table.
- b) If you play a three-note riff on Easy Level, what are the possible riffs that begin with a G? With a R? With a Y? Enter the total number of riffs in the table. How does this compare with your answers to a)?
- c) Considering your results in a) and b), try to predict how many possible combinations of notes you could play for a four-note riff, and for a five-note riff. Enter your predictions in the table. Explain your reasoning. Look for a pattern.



Number of notes	Number of riffs
1	3
2	
3	
4	
5	

Problem 3

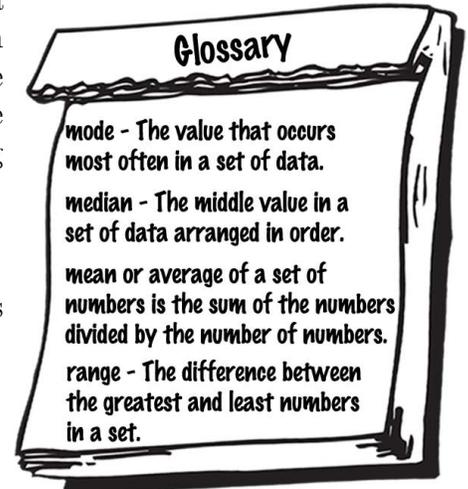


a) If a boy named Biff Smith can solve a certain problem, he will win one million dollars in cash, tax free. Here is the problem he must solve: A set of five single digit numbers has the following properties:

- the *mode* of the numbers is 1;
- the *mean (average)* of the numbers is 4;
- the *median* of the numbers is 5.

What are the five numbers?

b) Biff can win an extra \$100 000 if he can solve a second problem: Find a set of six whole numbers with a mean of 14.5, a mode of 1, and a range of 28. Is there more than one possible solution to this problem?



Problem 4

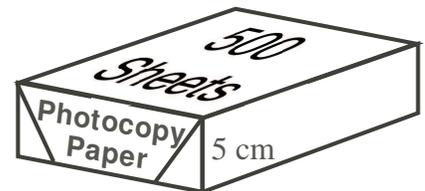
Biff tells the class he has won a million dollars, all in \$10 bills, and has them all stuffed in his backpack. Some of his classmates are a bit skeptical, and decide to investigate his claim through the following questions:



- If he had a million dollars, how many \$10 bills would Biff have?
- If he spent \$500 per week, for how many years would a million dollars last? (Assume a year is 52 weeks.)
- Assuming a sheet of photocopy paper is about the same thickness as a \$10 bill, estimate how high a stack of \$10 bills equivalent to a million dollars would be.

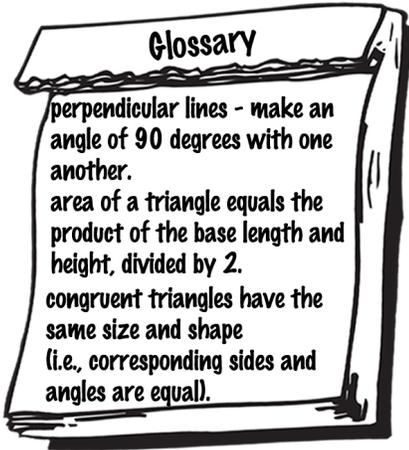
Extension :

- Would that quantity of \$10 bills fit in a backpack? (Each bill is 7 cm wide by 15 cm long, and about the same thickness as a sheet of photocopy paper.)

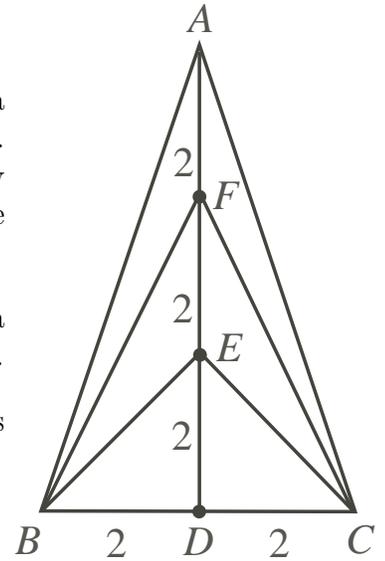


Problem 5

In the triangle ABC, AD is perpendicular to BC, and each of the line segments AF, FE, ED, BD, and DC has a length of 2 units.

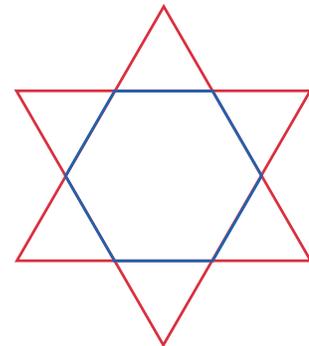


- a) Colour a section of the figure with area equal to $\frac{1}{2}$ the area of triangle ABC. Discover how to do this in as many different ways as you can, using the smaller triangles.
- b) Repeat part a) for regions with area equal to $\frac{1}{3}$ the area of triangle ABC.
- c) How many pairs of congruent triangles are there in this diagram?



Extension :

1. In the diagram at right, each line segment has the same length.
 - a) What type of triangles are on the exterior (outside) of the figure?
 - b) How many of these triangles are needed to fill the interior hexagon?



Problem 6: Packing up mp3 Players (For pairs or groups of students)

Liang needs to ship the new third generation mp3 players to her best friend Kim in Korea. Suppose the mp3 player has the following dimensions: height 12 cm, width 6 cm, and depth 1 cm.

- a) What is the total volume of the mp3 player in cubic cm? Cubic mm?
- b) Her friend Kim decides that she needs 24 mp3 players, one for each of her family members (and several friends) for Christmas. Design several boxes that Liang could use to send Kim all 24 mp3 players at once, assuming she packs them horizontally.
- c) Which of your boxes uses the least amount of cardboard? (Assume just a single layer of cardboard.)



Extension :

1. Repeat part c) without the constraint that the mp3 players must be packed horizontally. Can you find a smaller box this way? How do you know?

