

## Problem

### Packing up mp3 Players (For pairs or groups of students)

Liang needs to ship the new third generation mp3 players to her best friend Kim in Korea. Suppose the mp3 player has the following dimensions: height 12 cm, width 6 cm, and depth 1 cm.

- What is the total volume of the mp3 player in cubic cm? Cubic mm?
- Her friend Kim decides that she needs 24 mp3 players, one for each of her family members (and several friends) for Christmas. Design several boxes that Liang could use to send Kim all 24 mp3 players at once, assuming she packs them horizontally.
- Which of your boxes uses the least amount of cardboard? (Assume just a single layer of cardboard.)



#### **Extension :**

- Repeat part c) without the constraint that the mp3 players must be packed horizontally. Can you find a smaller box this way? How do you know?



**Hints**

**Hint 1** - a) How many cubic mm equal one cubic cm?

**Hint 2** - b) If the 24 mp3 players were packed in three stacks of 8, what would be the box dimensions? What other sets of stacks might work?

**Hint 3** - c) What are the lengths of the sides of each of the boxes you designed in part b)? How many pairs of identical sides are there?

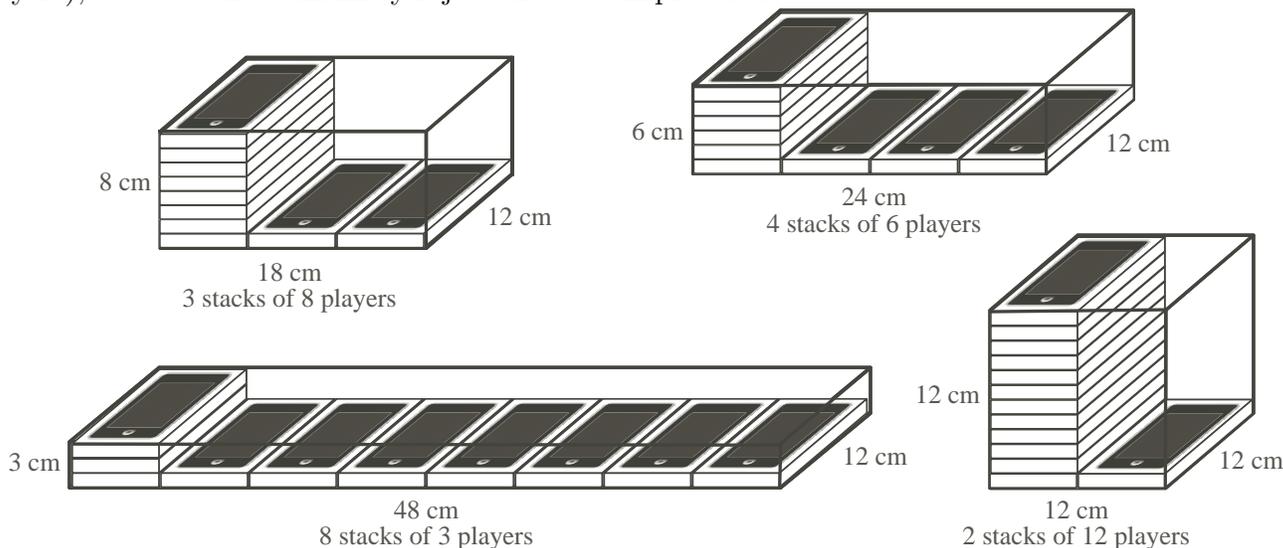
*Extension:*

**Hint 1** - If the 24 mp3 players were packed vertically in three rows of 8, what would be the box dimensions? In what other ways could you choose the rows?

## Solution

- a) The total volume of one mp3 player is  $12 \text{ cm} \times 6 \text{ cm} \times 1 \text{ cm} = 72$  cubic centimetres.
- b) The KEY IDEA is to make the dimensions of the base of the box some multiple of the 6 cm width and 12 cm height of a single mp3 player (since they are to be packed horizontally), so as not to have any empty space inside the box. Then find how many mp3 players fit into a single layer, and hence how many layers are needed. Here are some sample possibilities (illustrated below):
1. A box with base 18 cm by 12 cm permits 3 mp3 players in each layer, and hence must be 8 cm high (3 stacks of 8 players).
  2. A box with base 24 cm by 12 cm permits 4 mp3 players in each layer, and hence must be 6 cm high (4 stacks of 6 players).
  3. A box with base 48 cm by 12 cm permits 8 mp3 players in each layer, and hence must be 3 cm high (8 stacks of 3 players).
  4. A box with base 12 cm by 12 cm permits 2 mp3 players in each layer, and hence must be 12 cm high (2 stacks of 12 players).

There are other possibilities (e.g., a box with base 12 cm by 6 cm, 24 cm high, for 1 stack of 24 mp3 players, or a box with base 48 cm by 36 cm, 2 cm high, for 12 stacks of 2 mp3 players), but students will likely reject these as impractical.



- c) The amount of cardboard required is equal to the surface area of the box. If the box has dimensions length ( $l$ ), width ( $w$ ), and height ( $h$ ) in cm, then the surface area is  $S = 2 \times l \times w + 2 \times w \times h + 2 \times l \times h$  square centimetres. For the sample boxes in part b), the surfaces areas are:
1.  $S = 2 \times 18 \times 12 + 2 \times 12 \times 8 + 2 \times 18 \times 8 = 912$  square centimetres;
  2.  $S = 2 \times 24 \times 12 + 2 \times 12 \times 6 + 2 \times 24 \times 6 = 1008$  square centimetres;
  3.  $S = 2 \times 48 \times 12 + 2 \times 12 \times 3 + 2 \times 48 \times 3 = 1512$  square centimetres;
  4.  $S = 2 \times 12 \times 12 + 2 \times 12 \times 12 + 2 \times 12 \times 12 = 864$  square centimetres;

For the impractical box,  $S = 2 \times 12 \times 6 + 2 \times 6 \times 24 + 2 \times 12 \times 24 = 1008$  square centimetres.

Thus we see that the box which is a cube requires the least amount of cardboard. (This is true for any rectangular box of fixed volume, even when the side lengths are not whole numbers, but the proof requires more advanced mathematics.)

*Suggestion:* You may wish to discuss with the class whether there are other possible boxes. (There are.) Because the side lengths must be whole numbers, and the base of the box must be some integer multiple of 12 cm by 6 cm (to have no empty space), with a total volume of  $24 \times 72$  cubic centimetres, there are only a limited number of possibilities.

*Extension:*

1. Since the 24 mp3 players will exactly fit in 2 vertical rows of 12 in a 12 cm by 12 cm by 12 cm box, the 'minimal' box will be the same as for horizontal packing. Note that this only works because a row of 12 mp3 players packed vertically is 6 cm by 12 cm by 12 cm, which is exactly the same as a column (stack) of 12 mp3 players packed horizontally, due to the thickness being 1 cm.