Problem

The number 58 is the sum of some consecutive whole numbers.

#1	#2	Sum
20	21	
21	22	
23	24	

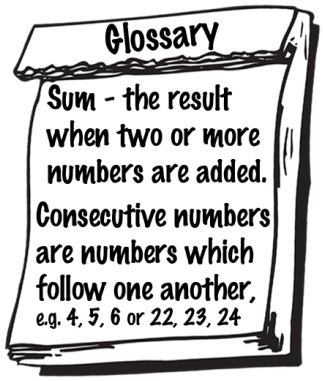
#1	#2	#3	Sum
16	17	18	
23	24	25	

a) Add the two numbers in each row of the given table. What do the sums have in common? Could 58 be the sum of **two** consecutive numbers? Why, or why not?

b) Add the **three** numbers in each row of the second table. What do the sums have in common? What consecutive numbers have a sum of 63?

c) Could 58 be the sum of three consecutive numbers? Why, or why not?

d) What are the consecutive numbers which have a sum of 58? (Are there 4 numbers which add to 58? 5 numbers? 6 numbers?)



***Extension* :**

1. Try to find another set of consecutive numbers which have a sum of 58.

Hints

Hint 1 - a) If you sum 2 consecutive numbers, what kind of number is the sum?

Hint 2 - b) What are the factors of each of the two sums in the second table?

Hint 3 - c) What does your result from b) tell you to check about 58?

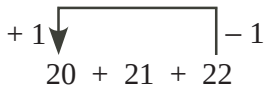
Hint 4 - d) Could 58 be the sum of four or more consecutive single digit numbers? Why, or why not?

Hint 5 - d) Could 58 be the sum of three or more consecutive numbers in the 20s? Why, or why not?

Solution

a) Each of the sums $20 + 21 = 41$, $21 + 22 = 43$, and $23 + 24 = 47$ is an odd number. This will be true of the sum of any two consecutive numbers. Thus, since 58 is even, it cannot be the sum of only two consecutive numbers.

b) Each of the sums $16 + 17 + 18 = 51$, and $23 + 24 + 25 = 72$ is divisible by 3 and is equal to 3 times the middle number ($51 = 3 \times 17$, and $72 = 3 \times 24$). This is true of any three consecutive numbers. For example, consider $20 + 21 + 22$. Before adding, we can subtract 1 from 22 and add it to 20 without changing the sum:



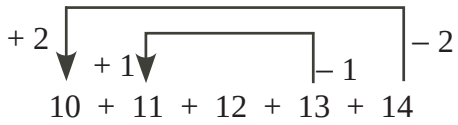
Thus we see that $20 + 21 + 22 = 21 + 21 + 21 = 3 \times 21 = 63$. Clearly we could do this with any three consecutive numbers. So any number which is the sum of three consecutive number must be divisible by 3.

c) Since 58 is not divisible by 3, it cannot be the sum of three consecutive numbers.
(Students may also discover this by systematic trials: $16 + 17 + 18 = 51$, $17 + 18 + 19 = 54$, $18 + 19 + 20 = 57$, $19 + 20 + 21 = 60$, so 58 is 'missing' in the set of sums.)

d) Trying for four consecutive numbers reveals that $58 = 13 + 14 + 15 + 16$.

***Extension* :**

1. Systematic trials of sums of five consecutive whole numbers give: $8 + 9 + 10 + 11 + 12 = 50$, $9 + 10 + 11 + 12 + 13 = 55$, $10 + 11 + 12 + 13 + 14 = 60 \ldots$ again, 58 is 'missing'. In fact, the sum of the five consecutive whole numbers is always divisible by 5, as can be the seen from the diagram for the example $10 + 11 + 12 + 13 + 14$:



Thus we see that $10 + 11 + 12 + 13 + 14 = 12 + 12 + 12 + 12 + 12 = 5 \times 12$, i.e., 5 times the middle number. We could do this with any such sum, so a sum of five consecutive whole numbers must be divisible by 5, which 58 is not.

The sum of six consecutive numbers consists of the sum of two sets of three consecutive numbers, hence must be divisible by 3, as above. So 58 cannot be the sum of six consecutive numbers.

A similar approach shows that the sum of seven consecutive whole numbers equals 7 times the middle number. A similar approach shows that 58 cannot be the sum of eight, nine, or ten whole numbers. And since the smallest sum of 11 whole numbers is $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 = 66$, we've run out of possibilities. Thus 58 can only be written as the sum of four consecutive whole numbers, as above.