Part II: For the Teacher

Curriculum Areas

Problem 1 - Number Sense, Algebra
Problem 2 - Number Sense
Problem 3 - Measurement, Pattern/Algebra
Problem 4 - Data Management, Measurement
Problem 5 - Data Management
Problem 6 - Geometry

Hints and Suggestions:

Problem 1

Hint 1 - Could Jasmina have 3 red boxes? Could she have 4 blue boxes?

Problem 2

Hint 1 - Which of the digits 0, 1, 2, ..., 9 are prime numbers?

Hint 2 - Could my ones digit be 5?

Extension:

Hint 1 - What are the possible choices for the last two digits?

Problem 3 b)

Hint 1 - How many texts can you send under the old plan for $25 total cost?

Problem 4

Hint 1 - Make a tree to see how many combinations exist.

Problem 5

Suggestion: Before starting this problem, discuss with the class how the climographs are constructed, emphasizing that the units for precipitation (mm) are distinct from the units for temperature (°C). Ask students to name the two different types of graphs that are used.

Hint 1 - d) How does the temperature affect the precipitation?

Problem 6

Suggestion: If possible, allow students to try finding the various possible paths on a wooden or plastic model of this geometric solid.
Solutions

Problem 1

Jasmina has some red and some blue boxes, and she puts exactly 3 cards into each red box, and exactly 7 cards into each blue box. Since there are 27 cards in total, it must be true that

\[ 3 \times (\text{Number of red boxes}) + 7 \times (\text{Number of blue boxes}) = 27 \]

Let \( R \) = the number of red boxes, and \( B \) = the number of blue boxes, so the equation becomes

\[ 3R + 7B = 27 \]

Since \( R \) and \( B \) must be positive whole numbers, we need a multiple of 3, plus a multiple of 7 which sum to 27.

Multiples of 3: \( 3, 6, 9, 12, 15, 18, 21, 24, 27, ... \)

Multiples of 7: \( 7, 14, 21, ... \)

Summing these in pairs reveals that the only possibility is \( 6 + 21 = 27 \). thus Jasmina has 2 red boxes and 3 blue boxes, so \( 3 \times 2 + 7 \times 3 = 27 \).

Problem 2

The only prime digits are 2, 3, 5, and 7. Since subtracting my ones digit from my tens digit gives a difference greater than my ones digit, my ones digit can only be 2, or 3, and my tens digit might be 5 or 7. 53 doesn’t work, since \( 5 - 3 = 2 \), which is not greater than 3. Thus I must be 52, 72, or 73.

Extension:

If I am a three-digit number with the same properties, then my last two digits can only be 52, 72, or 73. But my first digit can be any of 2, 3, 5, or 7, so I could be any of 252, 272, 273, 352, 372, 373, 552, 572, 573, 752, 772, or 773.

Problem 3

a)

<table>
<thead>
<tr>
<th>No. of Texts</th>
<th>Total Monthly Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$15</td>
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<tr>
<td>10</td>
<td>$19</td>
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<td>20</td>
<td>$23</td>
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<tr>
<td>30</td>
<td>$27</td>
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<tr>
<td>40</td>
<td>$31</td>
</tr>
<tr>
<td>50</td>
<td>$35</td>
</tr>
</tbody>
</table>

Note that the data points form a straight line, shown in grey.
b) The cost for unlimited texting is the basic cost plus an additional $10, i.e., $15 + $10 = $25. In the table, 20 texts cost $23 and 30 texts cost $27, so the “break even” point is in the middle, 25 texts costing $25, the ‘unlimited texting’ cost. This is reflected on the graph with the dotted line at $25 hitting the plotted line at 25 texts. For more than 25 texts, the unlimited plan is the better option.

Alternative Solution: For $10, at 40¢ for each text, you can send $10 ÷ $0.40 = 25 texts. Thus, for more than 25 texts, the unlimited plan is cheaper.

Problem 4

a) Represent the different food choices as follows:

- H - hamburger, C - cheeseburger, V - veggie burger,
- F - french fries, O - onion rings,
- R - regular drink, L - large drink.

Creating the tree as shown at right, we see that there is a total of 12 different ways the boys could select their lunch items.

b) If each boy selects the same three items, each lunch will cost the same. Since Hakim has only $30, each lunch can cost at most $30 ÷ 4 = $7.50. Assigning costs to each of the choices reveals that the only combinations Hakim could afford are:

- hamburger + french fries + regular drink ($7.25);
- veggie burger + french fries + regular drink ($6.75);
- veggie burger + french fries + large drink ($7.25); or
- veggie burger + onion rings + regular drink ($7.25).

Problem 5

The climographs reveal that:

a) Toronto has the highest July temperature (about 23°C).

b) Winnipeg has the lowest January temperature (about -15°C).

c) Halifax has the most rain in April (about 118 mm).
   (This is the trickiest observation. Note that, while the scales for precipitation vary from 0 to 250 mm for the last four climographs, for Halifax the scale is from 80 to 240 mm and for Montreal, from 50 to 250 mm. Thus, if student just look at the height of each bar graph for April, they will get the wrong answer. This is an excellent opportunity for discussion about how graphs can mislead.)

d) Halifax is the city likely to get the most snow, as it has low winter temperatures combined with high precipitation.

e) Vancouver is likely to get the least amount of snow, as the winter temperatures are above freezing, on average.

Answers for parts f) and g) will vary. Any justified response is acceptable.
Problem 6

a) The solid is a triangular prism.

b) There are two possible paths which have minimum length of 2 cm, $ACF$ and $ADF$, each with 1 vertex.

c) There are two possible paths with maximum length 5 cm, $ADEBCF$ and $ACBEDF$, each with 4 vertices (not counting the end-points A and F).

d) Other paths are $ABEF$, $ADEF$, $ABEDF$, $ACBEF$.

e)

<table>
<thead>
<tr>
<th>Path</th>
<th>Length</th>
<th>Vertices</th>
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</thead>
<tbody>
<tr>
<td>$ACF$</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$ADF$</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$ABEF$</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$ADEF$</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$ABEDF$</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>$ACBEF$</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>$ADEBCF$</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>$ACBEDF$</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

Note that the number of vertices is always 1 less than the path length.

Extension:

The answer to part c) could now include three possible paths, each of length 6 cm, with 4 vertices, namely

$$ADEBACF, ABEDACF, and ABCADEF$$