Problem

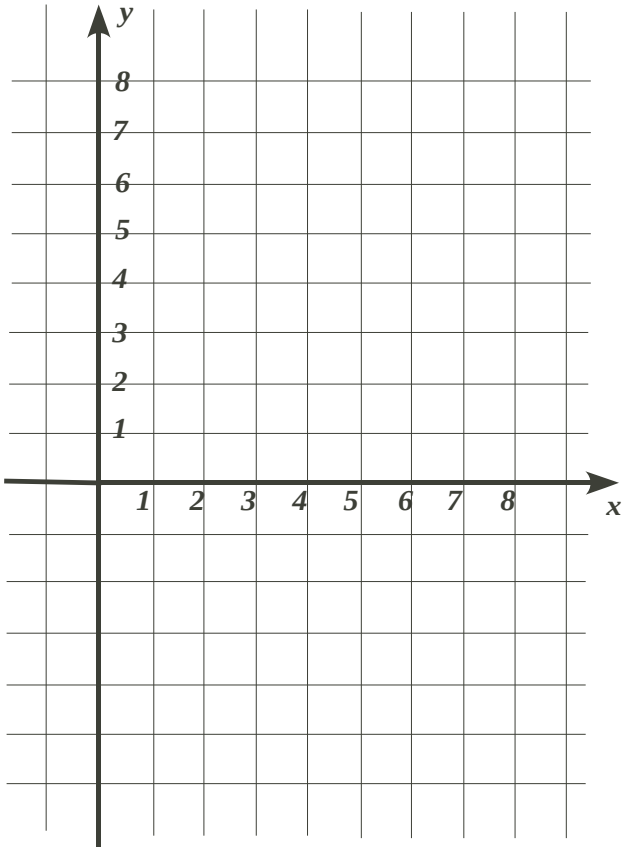
a) Plot the points $A(2, 2)$ and $B(6, 2)$ on the left graph below. If A and B are two consecutive vertices of a square, what is another pair of points, C and D, that would complete the square? Can you find more than one answer?

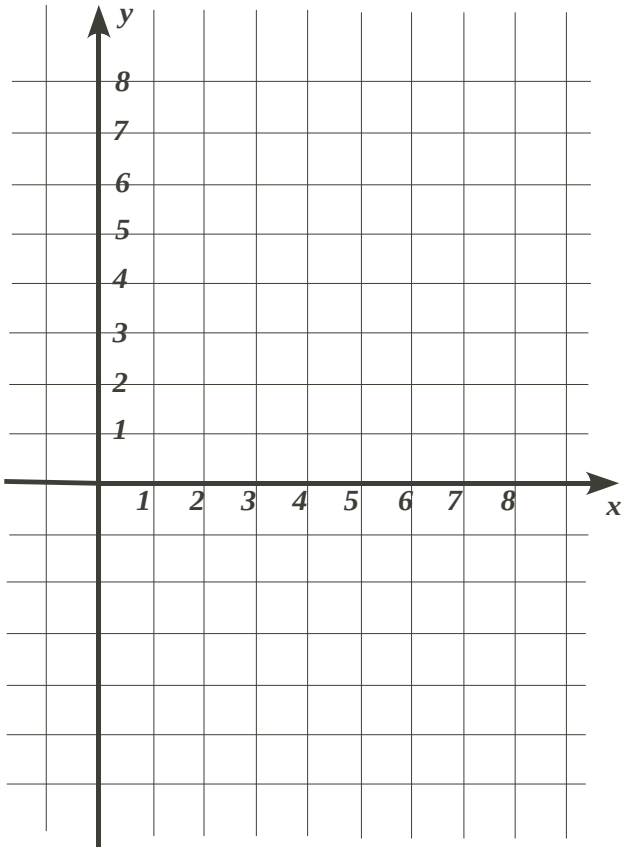
b) Plot the same two points A and B on the right graph below. If these points are two vertices of a right angled triangle, what would be the coordinates of the third vertex, C? Is there more than one answer?

c) If A and B are two consecutive vertices of a rectangle, how many other pairs of points C and D could be used to form a complete rectangle?

***Extension* :**

Suppose the point C in part b) is the third vertex of an equilateral triangle, rather than that of a right angled triangle. Locate the point C by construction (no need for coordinates).





Hints

Hint 1 - How far apart should adjacent vertices be to make a square?

Hint 2 - Do the other vertices need to be directly above A and B?

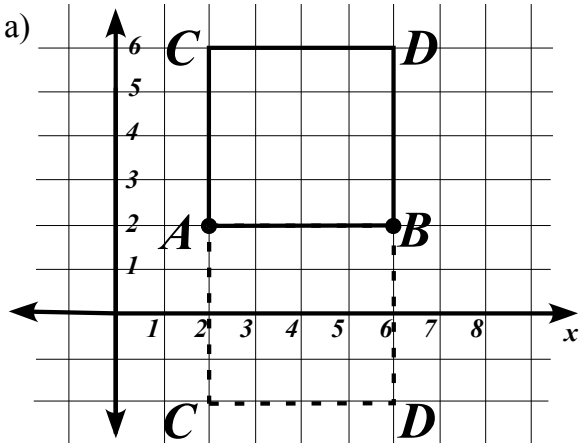
Hint 3 - Where could the right angle of the triangle be placed?

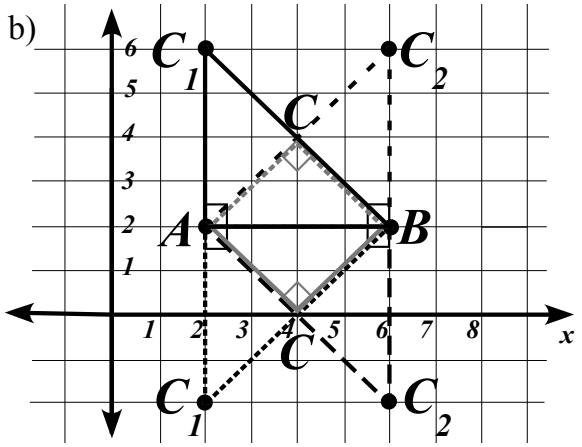
Extension:

Hint 1 - Would a compass be helpful?

Solution

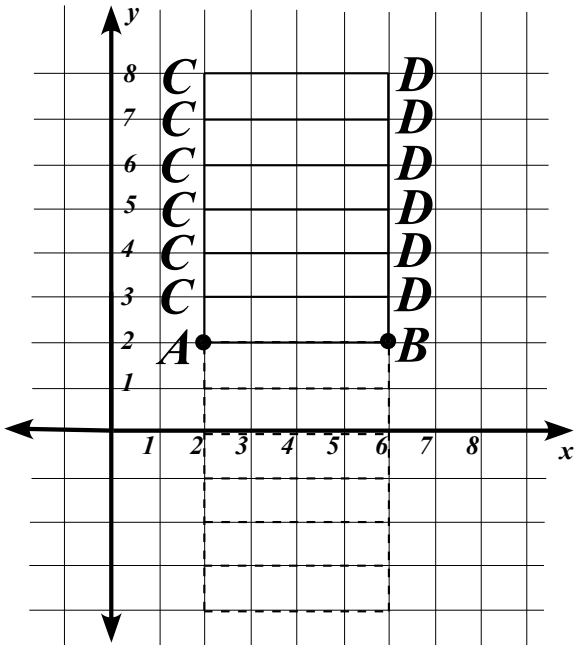
a), b) (See graph below.) Students may or may not realize that negative y-values could be used. A few students may recognize in part b) that C could be at (4, 4) or (4, 0).





c) Any pair of points $C(2, y)$ and $D(6, y)$ will work, for $y > 2$ or $y < 2$.

Students may suggest going beyong the range of 8 for y. They may also suggest the negative y possibilities.



Note: The roles of C and D may be reversed in parts a) and c).

Extension:

1. Using a compass, set its span to be the distance AB. Then draw arc 1 with A as the pivot point, and arc 2 with B as the pivot point. The intersection C of arcs 1 and 2 must be the same distance from both A and B. Thus ABC is an equilateral triangle.

   This construction could be repeated below AB.