Problem

a) Plot the points $A(2, 2)$ and $B(6, 2)$ on the left graph below. If $A$ and $B$ are two consecutive vertices of a square, what is another pair of points, $C$ and $D$, that would complete the square? Can you find more than one answer?

b) Plot the same two points $A$ and $B$ on the right graph below. If these points are two vertices of a right angled triangle, what would be the coordinates of the third vertex, $C$? Is there more than one answer?

c) If $A$ and $B$ are two consecutive vertices of a rectangle, how many other pairs of points $C$ and $D$ could be used to form a complete rectangle?

**Extension:**
Suppose the point $C$ in part b) is the third vertex of an equilateral triangle, rather than that of a right angled triangle. Locate the point $C$ by construction (no need for coordinates).
Hints

**Hint 1** - How far apart should adjacent vertices be to make a square?

**Hint 2** - Do the other vertices need to be directly above $A$ and $B$?

**Hint 3** - Where could the right angle of the triangle be placed?

**Extension:**

**Hint 1** - Would a compass be helpful?
Solution

a), b) (See graph below.) Students may or may not realize that negative $y$-values could be used. A few students may recognize in part b) that $C$ could be at $(4, 4)$ or $(4, 0)$.

c) Any pair of points $C(2, y)$ and $D(6, y)$ will work, for $y > 2$ or $y < 2$.

Students may suggest going beyond the range of 8 for $y$. They may also suggest the negative $y$ possibilities.

Note: The roles of $C$ and $D$ may be reversed in parts a) and c).
Extension:

1. Using a compass, set its span to be the distance \( AB \). Then draw arc 1 with \( A \) as the pivot point, and arc 2 with \( B \) as the pivot point. The intersection \( C \) of arcs 1 and 2 must be the same distance from both \( A \) and \( B \). Thus \( ABC \) is an equilateral triangle.

This construction could be repeated below \( AB \).