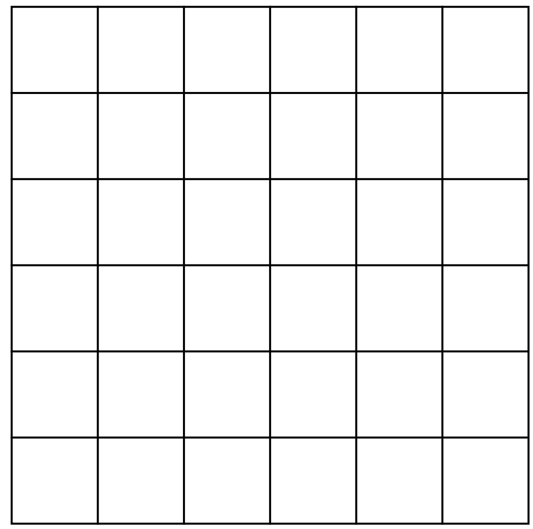
Problem

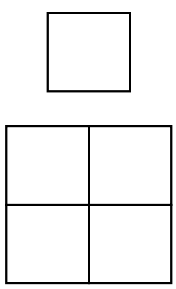
Betsy and Josh wish to find out how many squares of any size can be found in a grid that is 6 units by 6 units, as shown below. After trying to count all of the different-sized squares, they realize that it might be a good idea to try solving a simpler problem. (See the chart below.)



a) Instead of a 6×6 grid, what is the simplest possible grid? How many squares does it contain?

b) What are the next simplest dimensions for the grid? How many squares does it contain?

c) Complete the table below and look for a way to help Betsy and Josh solve the original problem. Write a sentence explaining how you found your answers.



Dimensions of the Grid	**Number of "Smallest" Squares**	**Total Number of Squares**
1×1	1	1
2×2	4	5
6×6		

Hints

Hint 1 - Are there squares of different sizes?

Hint 2 - In the 3×3 grid, how many 2×2 squares are there?

Hint 3 - How many more total squares are there in the 2×2 grid than in the 1×1? In the 3×3 grid than in the 2×2 gird?

Solution

a) The simplest grid is a single 1×1 grid.

b) The next simplest grid is a 2×2 grid with 1 large (2×2) square + 4 smallest (1×1) squares = 5 total squares.

c) The next simplest grid is a 3×3 grid with 1 (3×3 square) + 4 (2×2 squares) + 9 (1×1 squares) = 14 squares. Note that this equals the total number in the 2×2 grid (5), plus 3^2.

Similarly, the 4×4 grid has 1 (4×4 square) + 4 (3×3 squares) + 9 (2×2 squares) + 16 (1×1 squares) = 14 + 4^2 = 30 squares in total.

The pattern is: at each stage, add the number of 'smallest' (1×1) squares to the previous total to obtain the new total number of squares.

Note that the number of "smallest" squares in each case is just the square of the grid size

Dimensions of the Grid	**Number of "Smallest" Squares**	**Total Number of Squares**
1×1	1	1
2×2	4	5
3×3	9	14
4×4	16	30
5×5	25	55
6×6	36	91