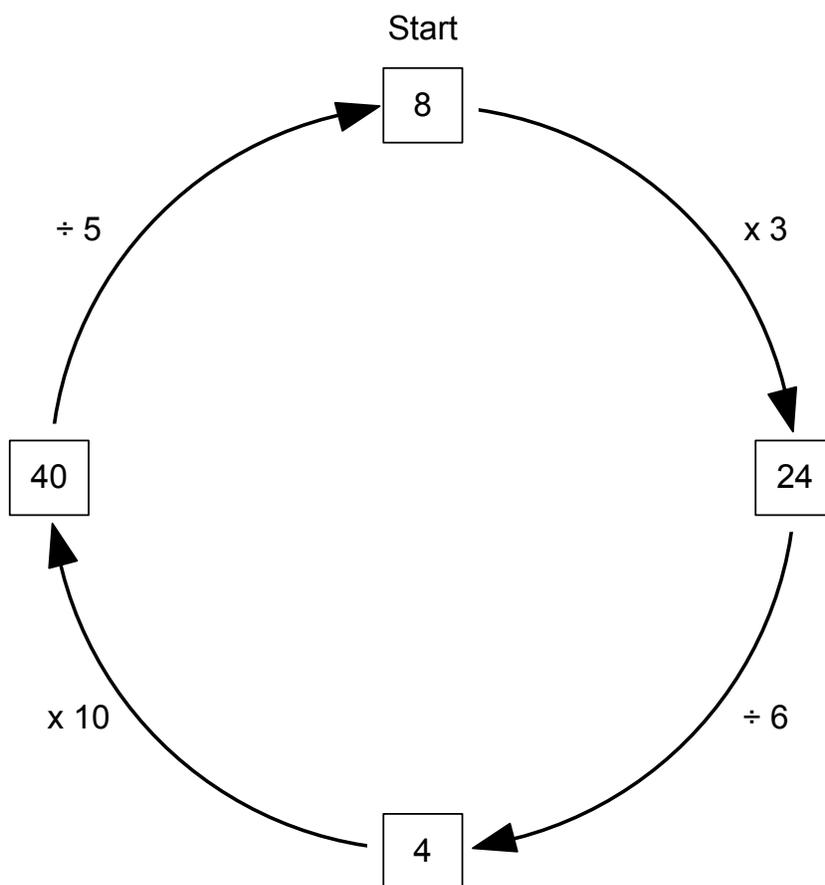# Problem of the Week
## Problem A and Solution
## Circle of Calculation

**Problem**

Determine the numbers that go in the boxes to make this diagram correct.

**Solution**

Since the circle started with the number 8, this is the solution.

Start

8

÷ 5

x 3

40

24

x 10

÷ 6

4

**Teacher's Notes**

Although there is only one correct solution using the starting number 8, this circle will work given any rational number in any one of the boxes as a starting value. In other words, no matter what number you start with, if you follow the operations in order, the application of the last operation will result in the starting number. For example, suppose you start with the number 50 in the box on the left side of the circle. The calculations would be as follows:

$$50 \div 5 = 10$$
$$10 \times 3 = 30$$
$$30 \div 6 = 5$$
$$5 \times 10 = 50$$

The circle will work with non-integer numbers as well. For example, if you start with the number 5 at the top of the circle, here are the calculations:

$$5 \times 3 = 15$$
$$15 \div 6 = \frac{5}{2}$$
$$\frac{5}{2} \times 10 = 25$$
$$25 \div 5 = 5$$

The reason this works is that multiplication and division are *inverse* functions. This means that if you multiply a number by some value and divide the result by the same value you end up with the original number. Also, if you divide a number by some value and then multiply the result by the same value you end up with the original number. (The exception here is if you try to divide a number by 0, since the result of dividing by 0 is *undefined*.)

It is a bit harder to explain why the circle works, because it involves more than one multiplication and division operation. We can see that overall we multiply by 3 and 10 which results in a multiple of 30 and we divide by 6 and 5 which results in a fraction of 30. However, we should be careful about the order in which we apply operations, especially division, The circle could be considered a *composition of functions*, which means we apply a function to the result of another function. It would be possible to show that the circle works by showing the result of a composition of four functions. Informally, we can see that the circle works by using a variable, rather than a number as a starting value. If we start with a variable $k$ at the top of the circle, here are the calculations:

$$k \times 3 = 3k$$
$$3k \div 6 = \frac{k}{2}$$
$$\frac{k}{2} \times 10 = 5k$$
$$5k \div 5 = k$$