Problem of the Week
Problem B and Solution
We've Got Your Number!

Problem

I am a nine-digit number. I contain each digit from 1 to 9 except the digit 8, and I contain two appearances of the digit 5. Discover what number I am by using the following clues.

- I am less than 500 000 000.
- My ten millions digit and my ones digit are the same.
- The sum of my hundred millions, ten millions, and millions digits is 18.
- My thousands digit is 1.
- My ten thousands digit is one more than my hundred thousands digit.
- My ones digit is equal to the sum of my hundreds digit and my tens digit.
- My hundreds digit is 3.

Solution

The mystery number is **459 671 325**.

This can be reasoned from the clues in the following steps.

- Since the digit 5 occurs twice and the ten millions digit and the units digit are the same, then they are both 5. The number looks like _ 5 _ _ _ _ _ _ 5

- Since the number is less than 500 000 000, the hundred millions digit must be 4 or less.

- Since the sum of the hundred millions, ten millions, and millions digits is 18, and the ten millions digit is 5, then the sum of the hundred millions digit and the millions digit must be 13. And since the hundred millions digit is 4 or less, the only combination that will work for the first 3 digits is 459. The number looks like 4 5 9 _ _ _ _ _ 5

- The hundreds digit is 3, and the 5 in the ones digit is the sum of the hundreds and tens digits; thus the tens digit is 2. The number now looks like 4 5 9 _ _ _ 3 2 5

- The thousands digit is 1. The number now looks like 4 5 9 _ _ 1 3 2 5

- All the digits have been used now except 6 and 7. Since the ten thousands digit is one more than the hundred thousands digit, they must be 7 and 6 respectively. The number we are looking for is 4 5 9 6 7 1 3 2 5

You may find it interesting to try this alternative approach. Start at the first clue with the greatest possible number, namely 497 655 321. Then work your way through the clues in order, adjusting this number to fit the given information. You'll discover that you have the answer after only five clues!