Problem of the Week
Problem C and Solution
Chillax

Problem
A rectangular container with base 9 cm by 11 cm has a height of 38.5 cm. Assuming that water expands 10% when it freezes, determine the depth to which the container can be filled so that when the contents freeze, the ice does not go above the top of the container.

Solution
Solution 1
To determine the volume of a rectangular solid, multiply the length, width and height. So the maximum volume of the container will be
\[ 9 \times 11 \times 38.5 = 3811.5 \text{ cm}^3. \]
Let the original depth of water in the container be \( h \) cm.
Then the water volume before freezing is \( 9 \times 11 \times h = (99 \times h) \text{ cm}^3. \) After the water freezes, the volume increases by 10% to 110% of its current volume. So after freezing, the volume will be
\[ 110\% \text{ of } 99 \times h = 1.1 \times 99 \times h = (108.9 \times h) \text{ cm}^3. \]
But the volume after freezing is the maximum volume, 3811.5 cm\(^3\). Therefore, \( 108.9 \times h = 3811.5 \) and it follows that \( h = 3811.5 \div 108.9 = 35 \) cm.
The container can be filled with water to a depth of 35 cm so that when it freezes the ice will not go over the top of the container.

Solution 2
In this solution we note that the length and width remain the same in both the volume calculation before and after the water freezes. We need only concern ourselves with the change in the depth of the water.
Let the original depth of water in the container be \( h \) cm.
After freezing, the depth increases by 10% to 110% of its depth before freezing. So after freezing, the depth will be 110% of \( h = 1.1 \times h = 38.5 \) cm, the maximum height of the container. Then \( h = 38.5 \div 1.1 = 35 \) cm.
The container can be filled with water to a depth of 35 cm so that when it freezes the ice will not go over the top of the container.