Problem of the Week
Problem C and Solution
A Fair Distribution

Problem

Four students share a $1000 prize for a project. They decide to split the money based on the amount of work each team member contributed to the project. Alison and Bob receive $\frac{1}{8}$ and $\frac{1}{5}$ of the total, respectively. Carl receives the average of what Alison and Bob receive. Dana receives the remainder. Determine the fraction of the prize money that Dana receives.

Solution

In the first solution, we solve the problem by working with the fractions and without calculating the amount of money each person receives. In the second solution, we determine the fraction of the winnings Dana receives by calculating the amount of money Dana receives and dividing by the total prize money.

Solution 1

Carl receives the average of $\frac{1}{8}$ and $\frac{1}{5}$ of the prize money.

Therefore, Carl receives $\dfrac{\frac{1}{8}+\frac{1}{5}}{2} = \dfrac{\frac{5}{40}+\frac{8}{40}}{2} = \dfrac{\frac{13}{40}}{2} = \dfrac{13}{80}$ of the total prize money.

Dana receives the remainder of the prize money.

Therefore, Dana receives $1 - \frac{1}{8} - \frac{1}{5} - \frac{13}{80} = \frac{80}{80} - \frac{10}{80} - \frac{16}{80} - \frac{13}{80} = \frac{41}{80}$ of the total prize money. Dana receives just over half of the prize money.

Solution 2

Alison receives $\frac{1}{8}$ of the prize money, so she receives $\frac{1}{8} \times \$1000 = \$125$.

Bob receives $\frac{1}{5}$ of the prize money, so he receives $\frac{1}{5} \times \$1000 = \$200$.

Carl receives the average of what Alison and Bob receive.

Therefore, Carl receives $\frac{\$125+\$200}{2} = \frac{\$325}{2} = \$162.50$.

Dana receives the remainder of the prize money.

Therefore, Dana receives $\$1000 - \$125 - \$200 - \$162.50 = \$512.50$.

That is, Dana receives $\frac{512.50}{1000} = \frac{5125}{10000} = \frac{41}{80}$ of the total prize money.

Dana receives just over half of the prize money.