Problem of the Week
Problem D and Solution
Concentrate

Problem
A large jug of lemonade contains a mixture of lemon juice concentrate and water. When 1 litre of water is added to the jug, the ratio, by volume, of juice concentrate to water is 1 : 2. When 1 litre of juice concentrate is added to the new mixture, the ratio becomes 2 : 3. Find the original ratio, by volume, of lemon juice concentrate to water in the jug.

Solution
Let \( j \) be the amount of juice, in litres, in the original mixture.
Let \( w \) be the amount of water, in litres, in the original mixture.

When 1 litre of water is added, the ratio of juice concentrate to water is 1 : 2. This tells us that
\[
\frac{j}{w + 1} = \frac{1}{2} \quad (1)
\]
Simplifying (1), we obtain \( w + 1 = 2j \) and \( w = 2j - 1 \) follows. (2)

When 1 litre of juice concentrate is added to the new mixture, the ratio becomes 2 : 3. This tells us that
\[
\frac{j + 1}{w + 1} = \frac{2}{3} \quad (3)
\]

From (2), since \( w = 2j - 1 \), we can substitute for \( w \) in (3), obtaining:
\[
\frac{j + 1}{2j - 1 + 1} = \frac{2}{3} \quad (4)
\]
\[
\frac{j + 1}{2j} = \frac{2}{3} \quad (5)
\]
\[
2(2j) = 3(j + 1) \]
\[
4j = 3j + 3 \]
\[
j = 3
\]

Substituting \( j = 3 \) in \( w + 1 = 2j \) we obtain \( w + 1 = 6 \) and \( w = 5 \) follows.

So there are 3 litres of juice concentrate in the container and 5 litres of water. The ratio of juice concentrate to water is 3 : 5.