



Problem of the Week

Problem D and Solution

All Aboard!

Problem

The captain of an Alaskan cruise was asked by one of the passengers how many guests were on board. The captain replied: $\frac{1}{6}$ of our guests are seniors, $\frac{1}{4}$ of our guests are children or teenagers, there are three times as many adults as teenagers, and there are 138 children on board. How many passengers are on board the cruise ship?

Solution

Let n be the total number of passengers.

Let s be the number of seniors, a be the number of adults, t be the number of teenagers, and c be the number of children.

Therefore, $n = s + a + t + c$.

Since $\frac{1}{6}$ of the passengers are seniors, $s = \frac{1}{6}n$.

Since $\frac{1}{4}$ of the passengers are children or teenagers, $c + t = \frac{1}{4}n$.

It is given that $c = 138$. Therefore, this becomes $138 + t = \frac{1}{4}n$, or $t = \frac{1}{4}n - 138$.

Since there are three times as many adults as teenagers, $a = 3t = 3(\frac{1}{4}n - 138)$.

Substituting into $n = s + a + t + c$, we have

$$\begin{aligned}n &= \left(\frac{1}{6}n\right) + 3\left(\frac{1}{4}n - 138\right) + \left(\frac{1}{4}n - 138\right) + 138 \\n &= \frac{1}{6}n + \frac{3}{4}n - 414 + \frac{1}{4}n - 138 + 138 \\n &= \frac{1}{6}n + n - 414 \\n &= \frac{7}{6}n - 414\end{aligned}$$

Rearranging, we have $\frac{7}{6}n - n = 414$, and so $\frac{1}{6}n = 414$ or $n = 2484$.

Therefore, there are 2484 passengers on board the cruise ship.

Although not required, we can determine the number of adults is 1449, the number of seniors is 414, the number of teenagers is 483. We can use this to verify the given information.

