



Problem of the Week

Problem E and Solution

Recovering Costs

Problem

In August, September and October, POTW Motors paid a constant price of \$1.25 per litre of fuel. In September, they used $x\%$ more litres of fuel than in August, where x is some positive number. In October, they used $x\%$ less litres of fuel than in September. The bookkeeper for POTW Motors forgot to record how much the company paid for fuel in September. However, she does know that POTW Motors paid a total of \$3125 for fuel in August and \$3080 for fuel in October.

Determine POTW Motor's total fuel cost for September.

Solution

Since the company paid \$3125 for fuel in August, the amount of fuel used in August was $\frac{3125}{1.25} = 2500$ litres.

Since the company paid \$3080 for fuel in October, the amount of fuel used in October was $\frac{3080}{1.25} = 2464$ litres.

In September POTW motors used $x\%$ more litres of fuel than they used in August.

In other words, they used $2500 + 2500\left(\frac{x}{100}\right) = 2500\left(1 + \frac{x}{100}\right)$ litres of fuel.

In October they used $x\%$ less litres of fuel than they used in September.

In other words, they used $\left(2500\left(1 + \frac{x}{100}\right)\right) - \left(2500\left(1 + \frac{x}{100}\right)\right)\left(\frac{x}{100}\right) = 2500\left(1 + \frac{x}{100}\right)\left(1 - \frac{x}{100}\right)$ litres of fuel.

We know that POTW Motors used 2464 litres of fuel in October. Therefore,

$$\begin{aligned} 2500\left(1 + \frac{x}{100}\right)\left(1 - \frac{x}{100}\right) &= 2464 \\ \left(1 + \frac{x}{100}\right)\left(1 - \frac{x}{100}\right) &= \frac{616}{625} \\ 1 - \frac{x^2}{10000} &= \frac{616}{625} \\ \frac{x^2}{10000} &= \frac{9}{625} \\ x^2 &= 144 \\ x &= 12, \quad \text{since } x \geq 0 \end{aligned}$$

Therefore, in September POTW motors used $2500\left(1 + \frac{x}{100}\right) = 2500\left(1 + \frac{12}{100}\right) = 2800$ litres of fuel. Their total fuel cost was $2800 \text{ L} \times \$1.25/\text{L} = \3500 .

