



Problem of the Week

Problem B and Solution

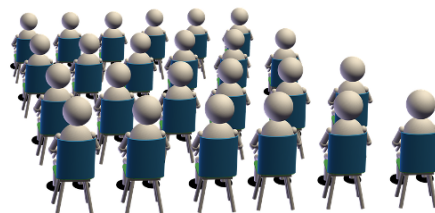
Saphyr Goes to a Concert

Problem

Saphyr and her friends are going to their local concert hall to see their favourite singer, Tinker Leisurely.

The rectangular hall is to be set up so that people sit in rows of chairs facing the stage.

Each row will have the same number of seats, with each row having at most 50 seats. The hall has a capacity for at most 115 rows. Saphyr has discovered that there are to be 4032 seats for the audience.



- Using what you know about the area of a rectangle and about the whole numbers that divide evenly into 4032, determine the possible configurations of seats in the hall.
- If the stage is 36 m wide and the chairs are to be spaced 1 m apart (from centre to centre), which of your answers in part (a) do you think gives the most reasonable dimensions for the seating? Why?

Solution

- We are looking for pairs of factors which have product 4032, thinking of the product as the number of seats in each row multiplied by the number of rows. We are given that each row must have at most 50 seats and we also know that there must be no more than 115 rows. Thus, one factor must be less than or equal to 50 and the other factor must be less than or equal to 115. The only possible such pairs with product 4032 are 36×112 , 42×96 , and 48×84 . Therefore, the hall could have 112 rows with 36 chairs in each row, or have 96 rows with 42 chairs in each row, or 84 rows with 48 chairs in each row.

Some students may list all possible products before checking constraints. They are:

1×4032 , 2×2016 , 3×1344 , 4×1008 , 6×672 , 7×576 , 8×504 , 9×448 , 12×336 ,
 14×288 , 16×252 , 18×224 , 21×192 , 24×168 , 28×144 , 32×126 , 36×112 , 42×96 ,
 48×84 , 56×72 , and 63×64 .

- Answers will vary.

Of the three possible arrangements, 112 rows of 36 seats would optimize the audience view of the stage, since the stage is 36 m wide. On the other hand, 84 rows of 48 seats would minimize the audience's distance from the stage, but give a less optimal viewing angle for those on the outer edges of the seating. Thus, a good compromise with reasonable stage view and distance may be 96 rows of 42 seats.

EXTENSION: If the rows did not have to have the same number of seats, how would your answer to part (b) change?