



Problem of the Week Problem C and Solution Again and Again

Problem

The fraction $\frac{1}{7}$ is equal to the repeating decimal $0.\overline{142857}$. Which digit occurs in the 2023rd place after the decimal point?

Solution

The digits after the decimal point occur in repeating blocks of the 6 digits 142857.

Since $\frac{2023}{6} = 337.1\overline{6} = 337\frac{1}{6}$, it follows that the 2023rd digit after the decimal point occurs after 337 complete repeating blocks of the 6 digits.

In 337 complete repeating blocks, there are $337 \times 6 = 2022$ digits in total. The 2023^{rd} digit is then the next digit. This corresponds to the first digit in the repeating block, which is 1.

Therefore, the digit 1 occurs in the 2023rd place after the decimal point.