

# Problem of the Week Problem D and Solution <br> How Much? 

## Problem

At a fundraiser hosted by a local restaurant, customers can pay as much or as little as they like for a meal, as long as they pay at least $\$ 1$. Any profits are donated to a local charity. One evening, the mean (average) price paid per customer was $\$ 55$. One more customer walked in and paid $\$ 70$ for a meal, bringing the average up to $\$ 56$. What is the highest possible price that a customer could have paid for their meal that evening? Do you think the amounts in your solution are reasonable for this situation?

## Solution

To calculate the mean (average) of a set of values, we first calculate the sum of the values in the set, and then divide that by the number of values in the set. It follows that the sum of the values in the set is equal to their average multiplied by the number of values in the set.

Let $n$ represent the number of customers that evening. The total amount paid by all customers that evening was therefore 56 n . The final customer paid 70 dollars for their meal. Before this customer arrived, there were $(n-1)$ customers and they had paid a total of $(56 n-70)$ dollars. At that point, the average price paid per customer was 55 dollars. Using this information, we can write and solve the following equation.

$$
\begin{aligned}
\frac{56 n-70}{n-1} & =55 \\
56 n-70 & =55(n-1) \\
56 n-70 & =55 n-55 \\
n & =15
\end{aligned}
$$

Since $n=15$, it follows that there were 15 customers that evening, and the total amount paid by all customers was therefore $56 n=56(15)=840$ dollars.

To determine the highest possible price that a customer could have paid for their meal that evening, we will assume that 13 of the customers paid the lowest possible price of $\$ 1$. Then the remaining customer would have paid $840-13 \times 1-70=757$ dollars.

Therefore, the highest possible price that a customer could have paid for their meal that evening is $\$ 757$.

Since this is a fundraiser, $\$ 1$ is probably a very small amount and $\$ 757$ would be considered a very generous donation for a meal.

## Extension:

How would the answer change if no two customers paid the same amount?

