

# Problem of the Week <br> Problem D and Solution <br> Let's Dance 

## Problem

The student council at POTW High School is throwing a school dance. They want to give a welcome gift to each Grade 9 student that attends the dance.

Gifts-R-Us charges $\$ 1.00$ per gift. However, if they were to purchase the gifts at Gifts-R-Us, they would exceed their budget by $\$ 17$.

At Presents-4-U, they only charge $\$ 0.80$ per gift. At this price, the student council would have $\$ 5.00$ left over in their budget.
Determine the number of gifts the student council is planning to buy.

## Solution

## Solution 1

Let $n$ represent the number of gifts that the student council is planning to buy.
Since each gift at Gifts-R-Us costs $\$ 1.00$, the student council would spend $1 \times n=n$ dollars in total. If the student council were to purchase all of the gifts they want at Gifts-R-Us, they would be short $\$ 17$ dollars in their budget. Therefore, the amount they have in their budget is $(n-17)$ dollars.
Since each gift at Presents-4-U costs $\$ 0.80$, the student council would spend $0.8 \times n=0.8 n$ dollars in total. If the student council were to purchase all of the gifts they want at Presents-4-U, they would have $\$ 5$ dollars left over in their budget. Therefore, the amount they have in their budget is $(0.8 n+5)$ dollars.

We have two expressions for the amount in their budget, so we can establish the equality $n-17=0.8 n+5$. This simplifies to $0.2 n=22$. After dividing each side by 0.2 , we obtain $n=110$.

Therefore, the student council is planning to buy 110 gifts.

## Solution 2

Let $n$ represent the number of gifts that the student council is planning to buy. Let $x$ represent the amount that the student council has budgeted.
Since the difference between the costs of a single gift is $\$ 1.00-\$ 0.80=\$ 0.20$, the total cost difference of buying $n$ gifts would be $\$ 0.2 n$.
To purchase from Gifts-R-Us, the student council would need to spend $\$ 17$ more than they budgeted. Therefore, they would need $(x+17)$ dollars. To purchase
from Presents-4-U, the student council would need to spend $\$ 5$ less than they budgeted. Therefore, they would need $(x-5)$ dollars. The total cost difference of purchasing $n$ gifts would be $(x+17)-(x-5)=22$ dollars.
We have two expressions for the cost difference and can establish the equality $0.2 n=22$. After dividing each side by 0.2 , we obtain $n=110$.
Therefore, the student council is planning to buy 110 gifts.
Note that in Solution 1 and Solution 2, we were able to solve for the number of gifts without calculating the budget. In Solution 3, we will first calculate the budget and then use that to calculate the number of gifts.

## Solution 3

Let $n$ represent the number of gifts that the student council is planning to buy. Let $x$ represent the amount that the student council has budgeted.

Since each gift at Gifts-R-Us costs $\$ 1.00, n$ gifts would cost $n \times \$ 1=\$ n$. Also, the student council would need to spend $\$ 17$ more than they budgeted.
Therefore, we have

$$
\begin{equation*}
n=x+17 \tag{1}
\end{equation*}
$$

Since each gift at Presents-4-U costs $\$ 0.80, n$ gifts would cost $n \times \$ 0.8=\$ 0.8 n$. Also, the student council would need to spend $\$ 5$ less than they budgeted. Therefore, we have

$$
\begin{equation*}
0.8 n=x-5 \tag{2}
\end{equation*}
$$

Substituting equation (1) into equation (2), we have

$$
\begin{aligned}
0.8 n & =x-5 \\
0.8(x+17) & =x-5 \\
0.8 x+13.6 & =x-5 \\
18.6 & =0.2 x \\
x & =93
\end{aligned}
$$

Thus, the student council has budgeted $\$ 93$.
Then, using equation (1), we see that $n=x+17=93+17=110$.
Therefore, the student council is planning to buy 110 gifts.

