Problem of the Week
Problem D and Solution
Missing the Fives II

Problem

Bobbi lists the positive integers, in order, excluding all multiples of 5. Her resulting list is

$$1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, \ldots$$

If the nth integer in Bobbi's list is 2023, what is the value of n?

Solution

Solution 1

Note that 2023 is two integers before 2025, which is a multiple of 5.
Beginning at 1, 2025 is the 405th multiple of 5, since $\frac{2025}{5} = 405$. That is, the integers from 1 to 2025 contain 405 groups of 5 integers.
Each of these 405 groups contain one integer that is a multiple of 5, and so Bobbi leaves out 406 integers (including 2024) in the list of all integers from 1 to 2025.
If the nth integer in Bobbi's list is 2023, then $n = 2025 - 406 = 1619$.

Solution 2

Note that 2023 is two integers before 2025, which is a multiple of 5.
Beginning at 1, 2025 is the 405th multiple of 5, since $\frac{2025}{5} = 405$. That is, the integers from 1 to 2025 contain 405 groups of 5 integers.
In Bobbi's list of integers, she leaves out the integers that are multiples of 5, and so in every group of five integers, Bobbi lists four of these integers. However, she also does not list 2024. Thus, if the nth integer in Bobbi's list is 2023, then $n = 405 \times 4 - 1 = 1619$.