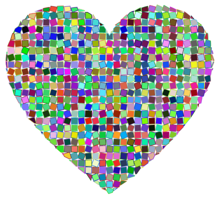
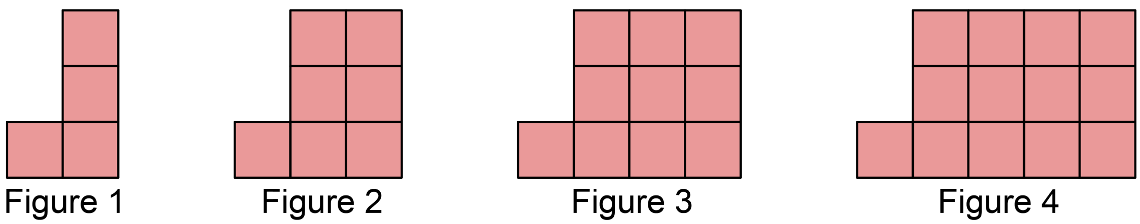


Problem of the Week
Problem A and Solution
Messy Mosaic

Problem

A pattern of figures is made from square blocks. Here are the first four figures of the pattern:



(a) Describe a *pattern rule* for the pattern.

(b) Using your pattern rule from part (a), what would Figure 10 be in the pattern?

(c) Using your pattern rule from part (a), how many blocks are in Figure 100?

Solution

(a) Here is one possible pattern rule. Each figure has a single column that is one block high, followed by columns that are three blocks high. Each figure of the pattern has one more three-block column than the previous figure.

(b) Using the pattern we described in part (a), we could draw pictures of the fifth through ninth figures before drawing the tenth figure. However, we notice that Figure 1 has 1 three-block column, Figure 2 has 2 three-block columns, Figure 3 has 3 three-block columns, and Figure 4 has 4 three-block columns. From this, we can extrapolate that Figure 10 has 10 three-block columns.

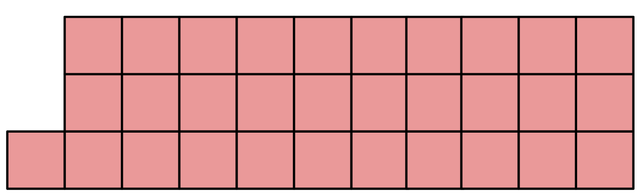


Figure 10

(c) Using the same logic as in part (b), there will be 100 three-block columns in Figure 100, giving a total of $3 \times 100 = 300$ blocks. When we include the first column in our count, this means there are 301 blocks in Figure 100.

Teacher's Notes

This problem provides a visual example of an *arithmetic sequence*. An arithmetic sequence is a sequence of numbers in which each number after the first is obtained from the previous number by adding a constant, called the common difference. In this problem, the common difference is 3.

The mathematical expression we can use to describe the general form of the n^{th} term in an arithmetic sequence is

$$a_n = a_1 + (n-1)d$$

where a_1 is the first term of the sequence, d is the common difference between pairs of numbers in the sequence, and a_n is the n^{th} term of the sequence.

In this problem, the first term of the sequence is 4 and the common difference is 3, so the mathematical expression for the n^{th} term of the sequence is

$$a_n = 4 + (n-1)(3)$$

We can use this formula to calculate the number of blocks in Figure 10:

$$a_{10} = 4 + (10-1)(3) = 4 + (9)(3) = 4 + 27 = 31$$

We can also use this formula to calculate the number of blocks in Figure 100:

$$a_{100} = 4 + (100-1)(3) = 4 + (99)(3) = 4 + 297 = 301$$