



Problem of the Week

Problem B and Solution

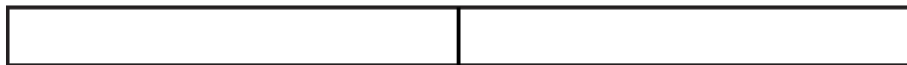
Wrecked Tangles

Problem

Gaby drew a rectangle and called it *Diagram 1*.



She then drew a rectangle divided into two equal parts, and called *Diagram 2*.



She then counted the total number of rectangles in *Diagram 2*. There is 1 rectangle on the left, 1 rectangle on the right, and the original whole rectangle, which makes 3 rectangles in total.

Gaby then drew a rectangle divided into three equal parts, called *Diagram 3*.



Gaby counted a total of 6 rectangles in *Diagram 3*. Can you confirm this?

- (a) Gaby continued drawing diagrams by dividing a rectangle into equal parts. *Diagram 4* is divided into four equal parts, *Diagram 5* is divided into five equal parts, and so on. Complete the table by determining the total number of rectangles in each diagram. Draw the diagrams to help you, and then look for a pattern in the total number of rectangles.

Diagram Number	Total Number of Rectangles
1	1
2	3
3	6
4	
5	
6	

- (b) Use the pattern you found in part (a) to predict the total number of rectangles in *Diagram 12*.



Solution

- (a) For each rectangle, we will assign the smallest rectangle a length of one unit.

Diagram 4 is a rectangle divided into 4 equal parts. In this diagram, there are 4 rectangles of length one unit, 3 of length two units, 2 of length three units, and 1 of length four units. This is a total of $4 + 3 + 2 + 1 = 10$ rectangles.



Diagram 5 is a rectangle divided into 5 equal parts. In this diagram, there are 5 rectangles of length one unit, 4 of length two units, 3 of length three units, 2 of length four units, and 1 of length five units. This is a total of $5 + 4 + 3 + 2 + 1 = 15$ rectangles.



Diagram 6 is a rectangle divided into 6 equal parts. In this diagram, there are 6 rectangles of length one unit, 5 of length two units, 4 of length three units, 3 of length four units, 2 of length five units, and 1 of length six units. This is a total of $6 + 5 + 4 + 3 + 2 + 1 = 21$ rectangles.



Now we see a pattern. The total number of rectangles for each diagram is equal to the sum of the diagram number and all the whole numbers smaller than it. Alternatively, the total number of rectangles for each diagram is equal to the diagram number plus the previous number of rectangles. So, the total number of rectangles in *Diagram 7* is equal to

$$7 + 6 + 5 + 4 + 3 + 2 + 1 = 28, \text{ or } 21 + 7 = 28.$$

- (b) Using the pattern from part (a), the total number of rectangles in *Diagram 12* is equal to $12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 78$, or $28 + 8 + 9 + 10 + 11 + 12 = 78$.