



Problem of the Week

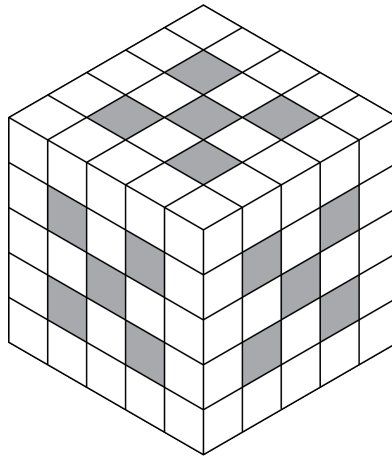
Problem C and Solution

Cubes Be Gone

Problem

A 5 by 5 by 5 cube is formed using identical 1 by 1 by 1 cubes.

Fifteen columns of cubes are then removed, five from front to back, five from top to bottom, and five from side to side. The columns that are removed are indicated by the shaded squares in the following diagram of the cube.



Following the removal of the fifteen columns of cubes, what percentage of the original number of 1 by 1 by 1 cubes remain?

Solution

Solution 1

In this solution, we analyze how many cubes are removed at each of the following stages: when removing the columns from front to back, when removing the columns from top to bottom, and finally when removing the columns from side to side.

When removing columns from the front to the back, 5 smaller cubes are removed from each layer. A total of 25 cubes are removed during this stage.

When removing cubes from top to bottom, since some cubes have already been removed with the front to back columns, the number of cubes removed from each layer is no longer the same. From top to bottom, the number of cubes removed from each layer is 5, 1, 4, 1, and 5. Thus, 16 additional cubes are removed during this stage.

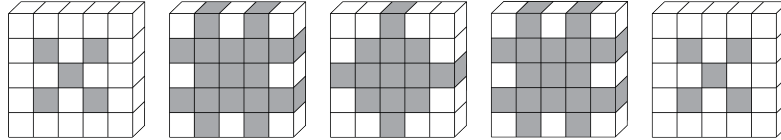
Finally, when removing cubes from side to side, the number of cubes removed at each layer is 5, 1, 4, 1, and 5, the same as the number removed in going from top to bottom. Thus, a total of 16 additional cubes are removed during this final stage.

The total number of cubes removed is $25 + 16 + 16 = 57$. The original 5 by 5 by 5 cube had $5 \times 5 \times 5 = 125$ of the smaller 1 by 1 by 1 cubes. Therefore, there are $125 - 57 = 68$ cubes remaining. The percentage of the original number of 1 by 1 by 1 cubes remaining after the removal of the fifteen columns is $68 \div 125 \times 100\% = 54.4\%$.



Solution 2

The diagram below shows each layer of the 5 by 5 by 5 cube after the columns of cubes have been removed. The cubes that have been removed are shaded in grey.



In the first layer, 20 of the 1 by 1 by 1 cubes remain. In the second layer, 8 of the 1 by 1 by 1 cubes remain. In the third layer, 12 of the 1 by 1 by 1 cubes remain. In the fourth layer, 8 of the 1 by 1 by 1 cubes remain. And in the final layer, 20 of the 1 by 1 by 1 cubes remain.

A total of $20 + 8 + 12 + 8 + 20 = 68$ of the 1 by 1 by 1 cubes remain. There were 125 1 by 1 by 1 cubes in the original 5 by 5 by 5 cube.

The percentage of the original number of 1 by 1 by 1 cubes remaining after the removal of the fifteen columns is $68 \div 125 \times 100\% = 54.4\%$.