Problem of the Week
Problem D and Solution
Making the Cut

Problem

Four line segments are drawn on a rectangular piece of paper, parallel to the top edge of the paper, as follows:

- Line segment A is 4 cm long and its left end touches the left edge of the paper.

- Line segment B is 3 cm long and its left end is 3 cm from the left edge of the paper.

- Line segment C is 6 cm long and its left end is 1.5 cm from the left edge of the paper.

- Line segment D is 4 cm long and its left end is 1 cm from the left edge of the paper.

The paper can be cut along a dotted line, L, parallel to the left edge of the paper and through each of the four line segments so that the total length of the pieces of the four line segments on each side of the cut is the same. What is the length, in cm, of the part of line segment A to the left of the cut?

Solution

Suppose the distance from the left edge of the paper to line L is d cm. Then the length of the part of line segment A to the left of the cut is d cm.

Since line segment B is 3 cm from the left edge of the paper, then the length of the part of line segment B to the left of the cut is $(d - 3)$ cm. Similarly, the lengths of the parts of line segments C and D to the left of the cut are $(d - 1.5)$ cm and $(d - 1)$ cm, respectively.
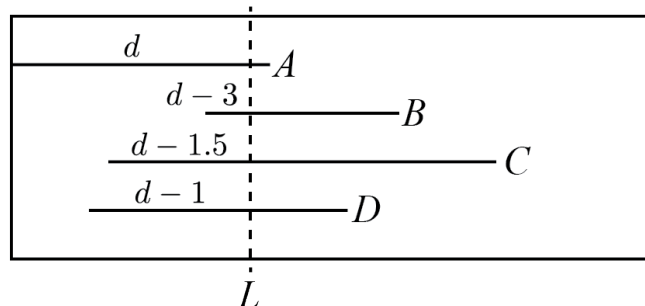
Therefore, the total length of the four line segments to the left of the cut is

$$d + (d - 3) + (d - 1.5) + (d - 1) = (4d - 5.5) \text{ cm.}$$

From here we proceed with two different solutions.

Solution 1

Since the total length of the four line segments on each side of the cut is the same, that means the total length of the four line segments on the left of the cut is equal to half of the total length of the four line segments, which is $\frac{1}{2}(4 + 3 + 6 + 4) = 8.5$ cm.
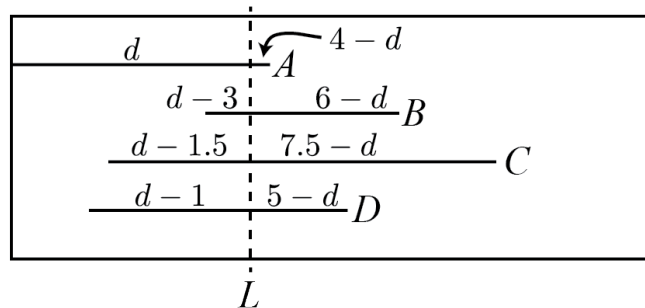
Thus, $4d - 5.5 = 8.5$, or $4d = 14$, or $d = 3.5$.

Therefore, the length of the part of line segment A to the left of the cut is 3.5 cm.

Solution 2

Since line segment A is 4 cm long and d cm of it is to the left of the cut, it follows that $(4 - d)$ cm of line segment A is to the right of the cut. Since line segment B is 3 cm long and $(d - 3)$ cm of it is to the left of the cut, it follows that $3 - (d - 3) = (6 - d)$ cm is to the right of the cut. Since line segment C is 6 cm long and $(d - 1.5)$ cm of it is to the left of the cut, it follows that $6 - (d - 1.5) = (7.5 - d)$ cm is to the right of the cut. Since line segment D is 4 cm long and $(d - 1)$ cm of it is to the left of the cut, it follows that $4 - (d - 1) = (5 - d)$ cm is to the right of the cut.

Therefore, the total length of the four line segments to the right of the cut is

$$(4 - d) + (6 - d) + (7.5 - d) + (5 - d) = (22.5 - 4d) \text{ cm}$$

Since the total length of the four line segments on each side of the cut is the same,

$$4d - 5.5 = 22.5 - 4d$$
$$8d = 28$$
$$d = 3.5$$

Therefore, the length of the part of line segment A to the left of the cut is 3.5 cm.