Problem of the Week

Problem D and Solution

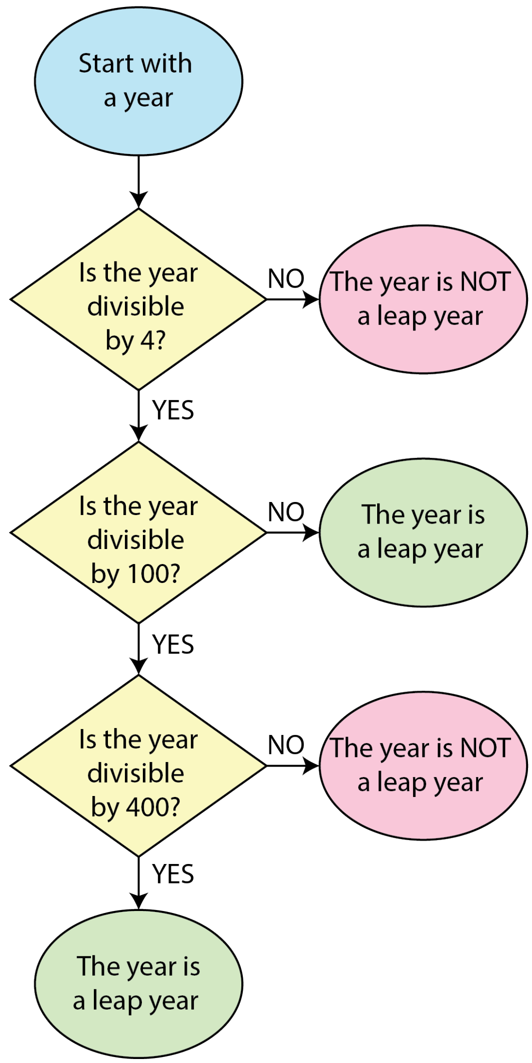
A Big Leap

Problem

Most people think of a year as 365 days, however it is actually slightly more than 365 days. To account for this extra time we use leap years, which are years containing one extra day.
Mara uses the flowchart shown to determine whether or not a given year is a leap year. She has concluded the following:

- 2018 was **not** a leap year because 2018 is not divisible by 4.
- 2016 was a leap year because 2016 is divisible by 4, but not 100.
- 2100 will **not** be a leap year because 2100 is divisible by 4 and 100, but not 400.
- 2000 was a leap year because 2000 is divisible by 4, 100, and 400.

If Mara chooses a year greater than 2000 at random, what is the probability that she chooses a leap year?



Solution

The probability of an event occurring is calculated as the number of favourable outcomes (that is, the number of outcomes where the event occurs) divided by the total number of possible outcomes. This is an issue in our problem because the number of years greater than 2000 is infinite. However, the cycle of leap years repeats every 400 years. For example, since 2044 is a leap year, so is 2444.

Thus, to determine the probability, we need to count the number of leap years in a 400-year cycle. From the flowchart we can determine that leap years are either

- multiples of 4 that are not also multiples of 100, or
- multiples of 4, 100, and 400.

Note that we can simplify the second case to just multiples of 400, since any multiple of 400 will also be a multiple of 4 and 100.

The number of multiples of 4 in a 400-year cycle is $\frac{400}{4} = 100$. However, we have included the multiples of 100, so we need to subtract these multiples. There are $\frac{400}{100} = 4$ multiples of 100 in a 400-year cycle. Thus, there are $100 - 4 = 96$ multiplies of 4 that are not multiples of 100. We now need to add back the the multiples of 400. There is $\frac{400}{400} = 1$ multiple of 400 in a 400-year cycle. Thus, there are $96 + 1 = 97$ numbers that are multiples of 4 and are not multiples of 100, or that are multiples of 400.

Therefore, for every 400-year cycle, 97 of these years will be a leap year.

Therefore, the probability of Mara choosing a leap year is $\frac{97}{400} = 0.2425$.