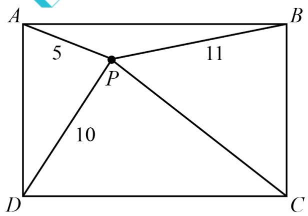


Problem of the Week

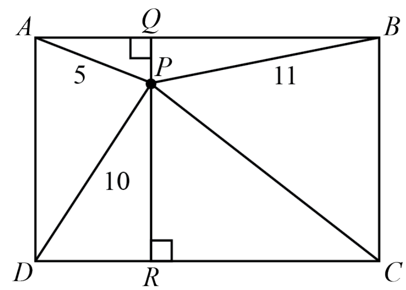
Problem D and Solution

Where is Pete?

Problem

Amir, Bita, Colin, and Delilah are standing on the four corners of a rectangular field, with Amir and Colin at opposite corners. Pete is standing inside the field 5 m from Amir, 11 m from Bita, and 10 m from Delilah. In the diagram, the locations of Amir, Bita, Colin, Delilah, and Pete are marked with A, B, C, D, and P, respectively. Determine the distance from Pete to Colin.

Solution



We start by drawing a line through P, perpendicular to AB and DC. Let Q be the point of intersection of the perpendicular with AB and R be the point of intersection with DC.
Since QP is perpendicular to AB, $\angle AQP = 90^\circ$ and $\angle BQP = 90^\circ$. Since PR is perpendicular to DC, $\angle DRP = 90^\circ$ and $\angle CRP = 90^\circ$. We also have that $AQ = DR$ and $BQ = CR$.

We can apply the Pythagorean Theorem in $\triangle AQP$ and $\triangle BQP$.

From $\triangle AQP$, we have $AQ^2 + QP^2 = AP^2 = 5^2 = 25$. Rearranging, we have

$$QP^2 = 25 - AQ^2 \quad (1)$$

From $\triangle BQP$, we have $BQ^2 + QP^2 = BP^2 = 11^2 = 121$. Rearranging, we have

$$QP^2 = 121 - BQ^2 \quad (2)$$

Since $QP^2 = QP^2$, from (1) and (2) we find that $25 - AQ^2 = 121 - BQ^2$ or $BQ^2 - AQ^2 = 96$. Since $AQ = DR$ and $BQ = CR$, this also tells us

$$CR^2 - DR^2 = 96 \quad (3)$$

We can now apply the Pythagorean Theorem in $\triangle DRP$ and $\triangle CRP$. From $\triangle DRP$, we have $DR^2 + RP^2 = DP^2 = 10^2 = 100$. Rearranging, we have

$$RP^2 = 100 - DR^2 \quad (4)$$

When we apply the Pythagorean Theorem to $\triangle CRP$ we have $CR^2 + RP^2 = CP^2$. Rearranging, we have

$$RP^2 = CP^2 - CR^2 \quad (5)$$

Since $RP^2 = RP^2$, from (4) and (5) we find that $100 - DR^2 = CP^2 - CR^2$, or

$$CR^2 - DR^2 = CP^2 - 100 \quad (6)$$

From (3), we have $CR^2 - DR^2 = 96$, so (6) becomes $96 = CP^2 - 100$ or $CP^2 = 196$. Thus $CP = 14$, since $CP > 0$.

Therefore the distance from Pete to Colin is 14 m.