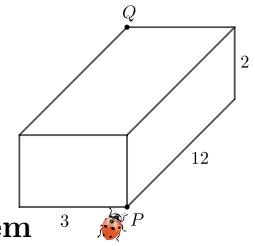


Problem of the Week
Problem E and Solution
Bug on the Outside

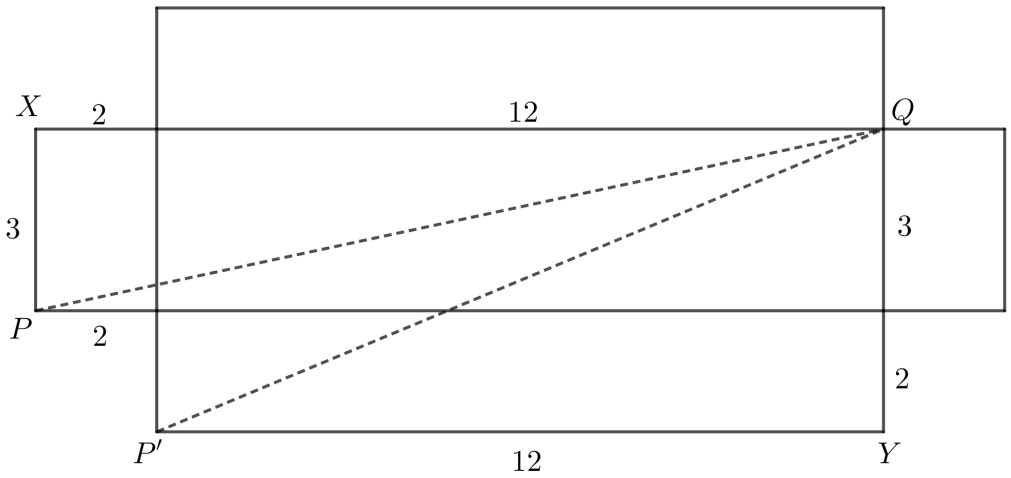
Problem

A ladybug walks on the surface of the 2 by 3 by 12 rectangular prism shown. The ladybug wishes to travel from P to Q.

What is the length of the shortest path from P to Q that the ladybug could take?

Solution

We fold out the sides of the prism so that they are laying on the same plane as the top of the prism. The diagram below shows the two-dimensional shape that results. As a result of folding out the sides, vertex P of the prism is a vertex of two different faces in the diagram. We call the second instance P'. We let X be the vertex adjacent to P along the side of length 3, and we let Y be the vertex adjacent to P' along the side of length 12.



The shortest distance for the ladybug to travel is in a straight line from P to Q or from P' to Q.

PQ is the hypotenuse of right-angled triangle PXQ. Using the Pythagorean Theorem,

$$PQ^2 = PX^2 + XQ^2 = 3^2 + 14^2 = 205$$

Thus, $PQ = \sqrt{205} \approx 14.3$, since $PQ > 0$.

$P'Q$ is the hypotenuse of right-angled triangle PYQ. Using the Pythagorean Theorem,

$$(P'Q)^2 = (P'Y)^2 + YQ^2 = 12^2 + 5^2 = 169$$

Thus, $P'Q = 13$, since $P'Q > 0$.

Since $P'Q < PQ$, the shortest distance for the ladybug to travel is 13 units on the surface of the block in a straight line from P' to Q.