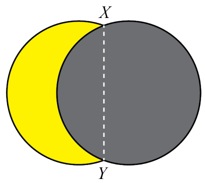


Problem of the Week

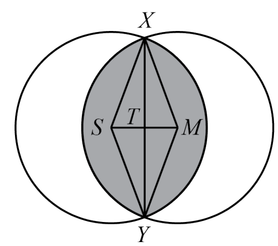
Problem E and Solution

Outside the Path of Totality

Problem

Yannick used a camera with a solar filter to capture a solar eclipse. From their location, the Moon blocked some, but not all of the Sun, so they saw a partial solar eclipse. When Yannick enlarged and printed the photo, they noticed that the distance represented by segment XY in the photo shown was 70 cm, and that the diameters of the Sun and the Moon were both 74 cm. Determine the percentage of the Sun that is blocked by the Moon in Yannick's photo, rounded to 1 decimal place.

Solution



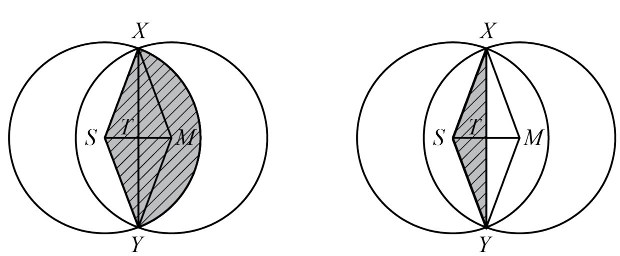
Let the centres of the Sun and Moon in the photo be S and M, respectively. We draw line segments SX, SY, MX, and MY. We draw line segment SM which intersects XY at T. The shaded region represents the portion of the Sun that is blocked by the Moon.

In our diagram we have drawn S and M inside the shaded region, but we need to prove they actually lie inside this region before we can proceed with the area calculation. Since each of the circles has radius $74 \div 2 = 37$, then $SX = SY = MX = MY = 37$. It follows that $\triangle SXY$ is isosceles. Similarly, $\triangle SXM$ and $\triangle SYM$ are congruent. Thus, $\angle XSM = \angle YSM$. Since $\triangle SXY$ is isosceles and SM bisects $\angle XSY$, then SM is perpendicular to XY at T, and $XT = TY = 70 \div 2 = 35$. By the Pythagorean Theorem in $\triangle STX$,

$$ST = \sqrt{SX^2 - XT^2} = \sqrt{37^2 - 35^2} = 12$$

Thus, $SM = 2 \times ST = 2 \times 12 = 24$. Since SM is smaller than the radius of the circles, it follows that S and M must lie inside the shaded region.

Now we can proceed with the area calculation. By symmetry, the area of the shaded region on each side of XY will be the same. The area of the shaded region on the right side of XY equals the area of acute sector XSY of the left circle minus the area of $\triangle SXY$. These areas are striped in the following diagrams.



First we find the area of $\triangle SXY$, which is $\frac{1}{2}(XY)(ST) = \frac{1}{2}(70)(12) = 420$.

Next we find the area of sector XSY. Using the cosine law in $\triangle SXY$,

$$\begin{aligned} XY^2 &= SX^2 + SY^2 - 2(SX)(SY)\cos(\angle XSY) \\ 70^2 &= 37^2 + 37^2 - 2(37)(37)\cos(\angle XSY) \\ 2162 &= -2738\cos(\angle XSY) \\ \angle XSY &= \cos^{-1}\left(-\frac{2162}{2738}\right) \approx 142.15^\circ \end{aligned}$$

Thus, the area of sector XSY is equal to $\frac{142.15^\circ}{360^\circ}\pi(37)^2$.

Then the area of the shaded region is equal to $2\left(\frac{142.15^\circ}{360^\circ}\pi(37)^2 - 420\right)$.

Finally, we calculate this area as a percentage of the area of the entire circle to obtain the following:

$$\frac{2\left(\frac{142.15^\circ}{360^\circ}\pi(37)^2 - 420\right)}{\pi(37)^2} \approx 59.4\%$$

It follows that the percentage of the Sun that is blocked by the Moon in Yannick's photo is equal to approximately 59.4%.