Problem of the Week

Problem E

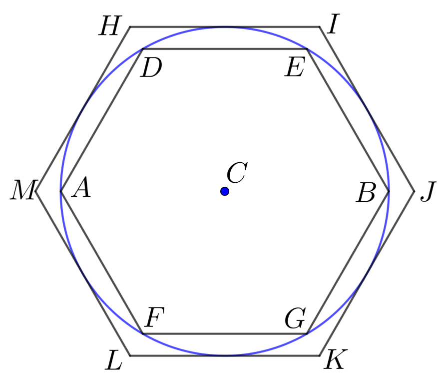
Pi Hexagons

Pi Day is an annual celebration of the mathematical constant π. Pi Day is observed on March 14, since 3, 1, and 4 are the first three significant digits of π.

Archimedes determined lower bounds for π by finding the perimeters of regular polygons inscribed in a circle with diameter of length 1. (An inscribed polygon of a circle has all of its vertices on the circle.) He also determined upper bounds for π by finding the perimeters of regular polygons circumscribed in a circle with diameter of length 1. (A circumscribed polygon of a circle has all sides tangent to the circle. That is, each side of the polygon touches the circle in one spot.)

In this problem, we will determine a lower bound for π and an upper bound for π by considering an inscribed regular hexagon and a circumscribed regular hexagon in a circle of diameter 1.

Consider a circle with centre C and diameter 1. Since the circle has diameter 1, it has circumference equal to π. Now consider the inscribed regular hexagon $DEBGFA$ and the circumscribed regular hexagon $HIJKLM$.

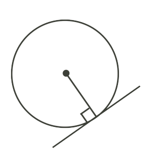


The perimeter of hexagon $DEBGFA$ will be less than the circumference of the circle, π, and will thus give us a lower bound for the value of π. The perimeter of hexagon $HIJKLM$ will be greater than the circumference of the circle, π, and will thus give us an upper bound for the value of π.

Using these hexagons, determine a lower and an upper bound for π.

NOTE: For this problem, you may want to use the following known results:

1. A line drawn from the centre of a circle perpendicular to a tangent line meets the tangent line at the point of tangency.



2. For a circle with centre C, the centres of both the inscribed and circumscribed regular hexagons will be at C.