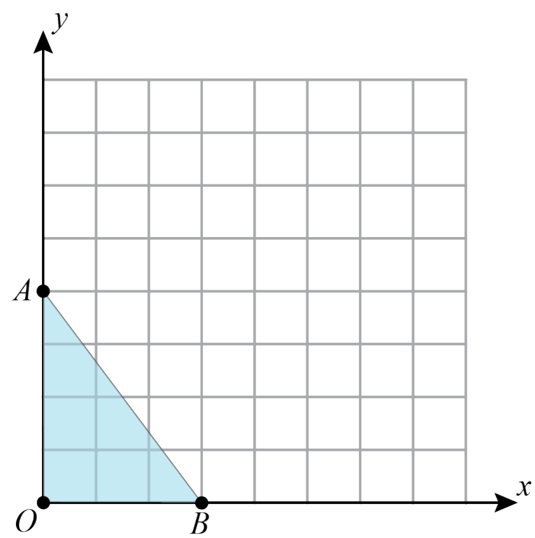
Problem of the Week
Problem B and Solution
Triangular Fun

Problem

Work through the parts that follow using the following coordinate plane, where grid lines are spaced 1 unit apart.



(a) Label the coordinates of the points A, O, and B.

(b) Plot point C on the y-axis so that OC is twice the length of OA. Then plot point D on the x-axis so that OD is twice the length of OB. Label the coordinates of points C and D.

(c) Show that the area of $\triangle COD$ is four times the area of $\triangle AOB$. To show this, you may use your diagram or an area formula.

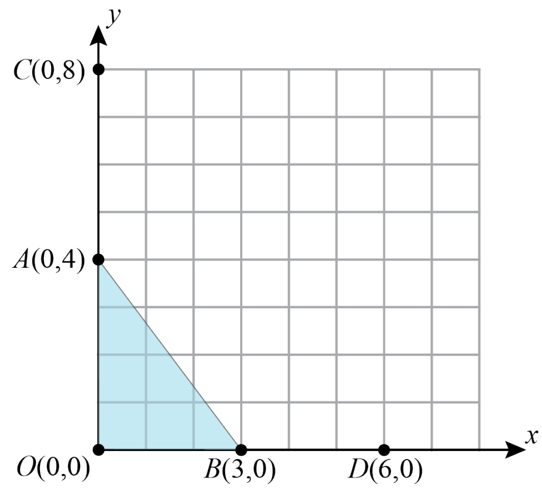
Extension: In general, if you double the lengths of the two perpendicular sides of any right-angled triangle, will the area of the new triangle be four times the area of the original triangle? Explain.

Not printing this page? You can use our interactive worksheet.

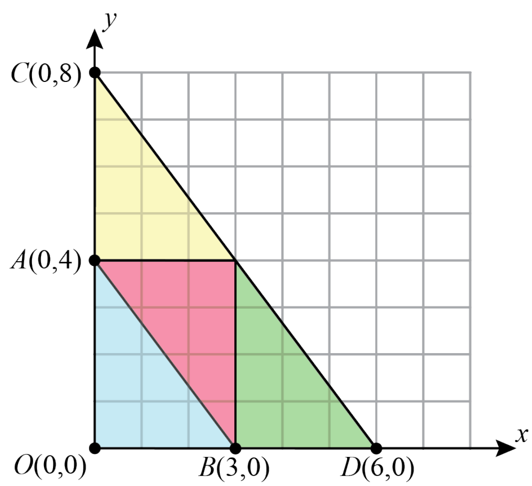
Solution

(a) The coordinates are $A(0,4)$, $O(0,0)$, and $B(3,0)$.

(b) Points C and D are plotted on the diagram, and their coordinates are $C(0,8)$ and $D(6,0)$, as shown.



(c) The diagram shows $\triangle COD$ divided into four smaller right-angled triangles, each congruent to $\triangle AOB$, with perpendicular sides of length 3 and 4. Therefore, the area of $\triangle COD$ is four times the area of $\triangle AOB$.



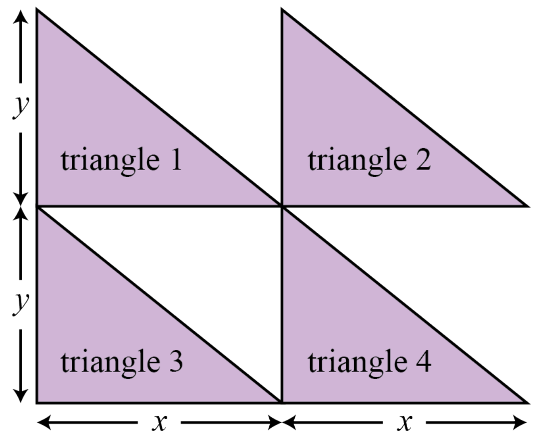
Alternatively, we can calculate the areas of $\triangle AOB$ and $\triangle COD$ using the area formula: Area = base $\times$ height $\div$ 2.

$$\begin{aligned}\text{Area of } \triangle AOB &= 3 \times 4 \div 2 \\ &= 12 \div 2 \\ &= 6\end{aligned} \qquad \begin{aligned}\text{Area of } \triangle COD &= 6 \times 8 \div 2 \\ &= 48 \div 2 \\ &= 24\end{aligned}$$

Since $6 \times 4 = 24$, the area of $\triangle COD$ is four times the area of $\triangle AOB$.

Extension Solution:

We will start with a right-angled triangle where the two perpendicular sides have lengths of x and y. We then create four copies of this triangle, numbered from 1 to 4, and arrange them as shown. The total area of the four triangles is four times the area of the original triangle.



Now, if we rotate triangle 2 by 180°, the four triangles will be in the shape of a larger right-angled triangle where the lengths of the two perpendicular sides are $2x$ and $2y$. Thus, if you double the lengths of the two perpendicular sides of any right-angled triangle, the area of the new triangle will be four times the area of the original triangle.