



Problem of the Week

Problem C and Solution

Stargazing

Problem

In a distant solar system, four different comets: Hypatia, Fibonacci, Lovelace, and Euclid, passed by the planet Ptolemy in 2023. On Ptolemy, it is known that the Hypatia comet appears every 3 years, the Fibonacci comet appears every 6 years, the Lovelace comet appears every 8 years, and the Euclid comet appears every 15 years.

When is the next year that all four comets will pass by Ptolemy?

Solution

Since the Hypatia comet appears every 3 years, it will pass by Ptolemy in the following numbers of years: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30....

Since the Fibonacci comet appears every 6 years, it will pass by Ptolemy in the following numbers of years: 6, 12, 18, 24, 30,....

Therefore, both the Hypatia and Fibonacci comets will pass by Ptolemy in the following numbers of years: 6, 12, 18, 24, 30,....

This happens because these numbers are *common multiples* of 3 and 6. If we want to determine when all four comets next pass by Ptolemy, we need to find the *least common multiple* (LCM) of 3, 6, 8, and 15. We shall do this in two ways.

Solution 1

The first way to find the LCM is to list the positive multiples of 3, 6, 8, and 15, until we find a common multiple in each list.

Number	Positive Multiples
3	3, 6, 9, 12, 15, 18, 21, ..., 108, 111, 114, 117, 120 , 123, ...
6	6, 12, 18, 24, 30, 36, 42, ..., 96, 102, 108, 114, 120 , 126, ...
8	8, 16, 24, 32, 40, 48, 56, ..., 104, 112, 120 , 128, ...
15	15, 30, 45, 60, 75, 90, 105, 120 , 135, ...

Thus, the LCM of 3, 6, 8, and 15 is 120. Therefore, the next time all four planets will pass by Ptolemy is in 120 years. This will be the year 2143.



Solution 2

The second way to determine the LCM is to rewrite 3, 6, 8, and 15 as a prime or a product of prime numbers. (This is known as *prime factorization*.)

- $3 = 3$
- $6 = 2 \times 3$
- $8 = 2 \times 2 \times 2$
- $15 = 3 \times 5$

The LCM is calculated by determining the greatest number of each prime number in any of the factorizations (here we will have three 2s, one 3, and one 5), and then multiplying these numbers together. This gives $2 \times 2 \times 2 \times 3 \times 5 = 120$. Therefore, the next time all four planets will pass by Ptolemy is in 120 years. This will be the year 2143.

NOTE: The second method is a more efficient way to find the LCM, especially when the numbers are quite large.