$A=B$

Problem of the Week
Problem C and Solution
Exponential Expressions

Problem

We are given two expressions:

$$\text{Expression } A\text{: } 72 \times 7^x$$

$$\text{Expression } B\text{: } 441 \times 2^y$$

Given that x and y are positive integers, find all ordered pairs (x, y) so that the value of Expression A is equal to the value of Expression B.

Solution

Solution 1

We write each expression as the product of prime numbers.

Expression $A = (2^3)(3^2)(7^x)$ and Expression $B = (3^2)(7^2)(2^y)$.

Since x and y are each positive integers and the expressions are equal in value, then the corresponding exponents for each prime number must be equal. Therefore, $x = 2$ and $y = 3$ is the only integer solution.

Thus, the only ordered pair is $(2, 3)$.

Solution 2

Setting the two expressions equal to each other, we have

$$72 \times 7^x = 441 \times 2^y$$

Dividing both sides by 9, we have

$$8 \times 7^x = 49 \times 2^y$$

Expressing each side of the equation as the product of prime numbers, we have

$$2^3 \times 7^x = 7^2 \times 2^y$$

Since x and y are each positive integers and the expressions are equal in value, then the corresponding exponents for each prime number must be equal. Therefore, $x = 2$ and $y = 3$ is the only integer solution.

Thus, the only ordered pair is $(2, 3)$.