



Problem of the Week

Problem C and Solution

A Small Leap

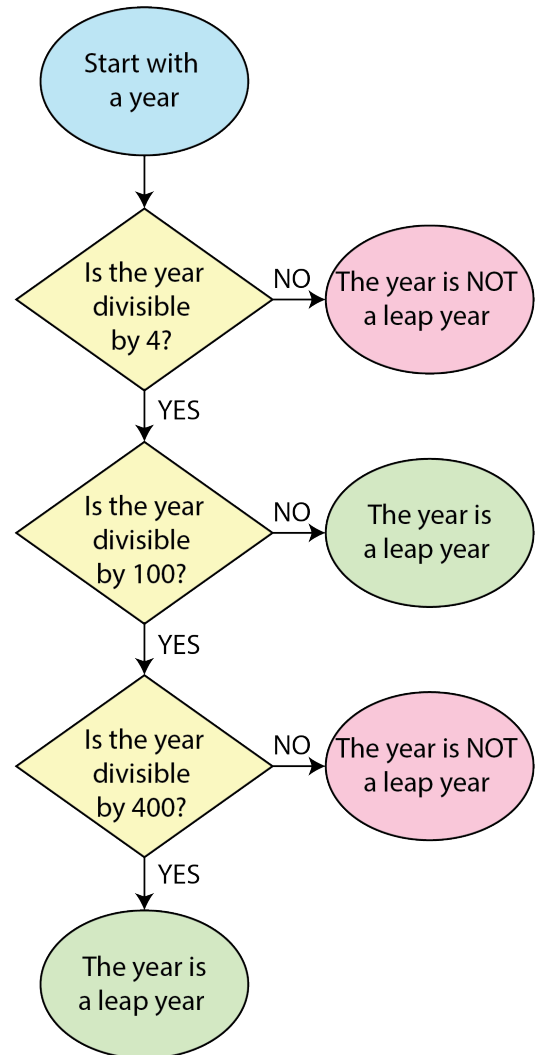
Problem

Most people think of a year as 365 days, however it is actually slightly more than 365 days. To account for this extra time we use leap years, which are years containing one extra day.

The flowchart shown can be used to determine whether or not a given year is a leap year. Using the flowchart, we can conclude the following:

- 2018 was **not** a leap year because 2018 is not divisible by 4.
- 2016 was a leap year because 2016 is divisible by 4, but not 100.
- 2100 will **not** be a leap year because 2100 is divisible by 4 and 100, but not 400.
- 2000 was a leap year because 2000 is divisible by 4, 100, and 400.

How many leap years are there between the years 2000 and 2400, inclusive?





Solution

From the flowchart we can determine that leap years are either

- multiples of 4 that are not also multiples of 100, or
- multiples of 4, 100, and 400.

Note that we can simplify the second case to just multiples of 400, since any multiple of 400 will also be a multiple of 4 and 100.

First we notice that 2000 and 2400 are both multiples of 400, so they are both leap years. In fact, they are the only multiples of 400 between 2000 and 2400, inclusive.

Next we count the multiples of 4 between 2000 and 2400, inclusive. Writing out some of the first few multiples of 4 gives: 2000, 2004, 2008, 2012, . . .

We will look at the 400 numbers from 2000 to 2399 and ignore 2400 for the moment since we already know it's a leap year. Since 2000 is a multiple of 4 and every fourth number after that is also a multiple of 4, it follows that $\frac{1}{4}$ of the 400 numbers from 2000 to 2399 will be multiples of 4. Thus, there are $\frac{1}{4} \times 400 = 100$ multiples of 4 between 2000 and 2399, inclusive.

However, we have included the multiples of 100, so we need to subtract these. These are 2000, 2100, 2200, and 2300. Thus there are $100 - 4 = 96$ multiples of 4 between 2000 and 2399, inclusive, that are not also multiples of 100.

Thus, in total, there are $2 + 96 = 98$ leap years between 2000 and 2400, inclusive.