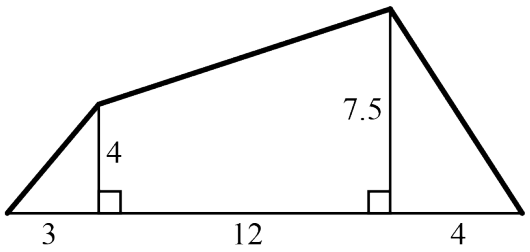
Problem of the Week
Problem C and Solution
Climbing Up and Down

Problem

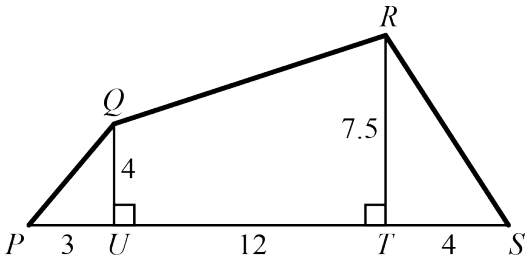
An obstacle course is created as follows. Two posts, the first being 4 m high and the second being 7.5 m high, are placed 12 m apart. A rope ladder starts on the ground, 3 m from the base of the first post, and finishes at the top of the first post. A second rope ladder connects the tops of the two posts. A third rope ladder starts at the top of the second post and finishes on the ground 4 m from the base of the second post. An illustration of the obstacle course is provided below.



To complete the obstacle course, Jesse has to climb along the three rope ladders. If each of the three rope ladders forms a straight line, then determine the total distance Jesse must travel on the rope ladders.

Solution

We begin by labelling the diagram.
Let P be where the first rope ladder meets the ground.
Let U be the base of the first post and Q be the top of the first post.
Let T be the base of the second post and R be the top of the second post.
Let S be where the third rope ladder meets the ground.



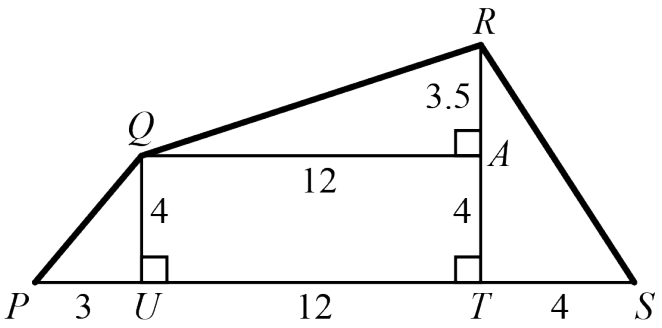
The total distance Jesse must travel on the rope ladders is equal to $PQ + QR + RS$.

First, we will calculate the lengths of PQ and RS.

Since $\angle PUQ = 90°$, we can apply the Pythagorean Theorem in $\triangle PUQ$. Thus, $PQ^2 = QU^2 + PU^2 = 4^2 + 3^2 = 16 + 9 = 25$. Therefore, $PQ = 5$, since $PQ > 0$.

Similarly, $\angle RTS = 90°$, so we can apply the Pythagorean Theorem in $\triangle RTS$. Thus, $RS^2 = RT^2 + TS^2 = 7.5^2 + 4^2 = 56.25 + 16 = 72.25$. Therefore, $RS = 8.5$, since $RS > 0$.

Now we will calculate the length of QR. Draw a line from Q perpendicular to RT. Let A be the point of intersection of the perpendicular with RT. Since QA is perpendicular to RT, $QATU$ is a rectangle. Therefore, $QA = UT = 12$ and $AT = QU = 4$. Thus, $AR = RT - AT = 7.5 - 4 = 3.5$.



Since $\angle QAR = 90°$, we can apply the Pythagorean Theorem in $\triangle QAR$. Thus, $QR^2 = QA^2 + AR^2 = 12^2 + 3.5^2 = 144 + 12.25 = 156.25$. Therefore, $QR = 12.5$, since $QR > 0$.

Therefore, the total distance Jesse must travel on the rope ladders is $PQ + QR + RS = 5 + 12.5 + 8.5 = 26$ m.