



Problem of the Week

Problem C and Solution

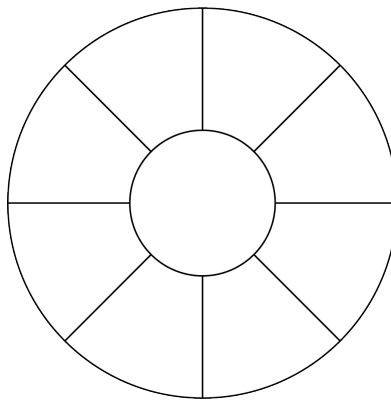
A Divided Circle

Problem

Two circles are said to be *concentric* if they have the same centre.

In the diagram below, there are two concentric circles. The radius of the smaller circle is 5 cm. The portion inside the larger circle that is outside of the smaller circle is divided into eight congruent pieces that each have the same area as the smaller circle.

In terms of π , what is the circumference of the larger circle?



Solution

Since the radius of the smaller circle is 5 cm, it follows that the area of this circle is $\pi(5)^2 = 25\pi \text{ cm}^2$.

To determine the circumference of the larger circle, we will first find its radius.

Let r be the radius of the larger circle. Then the area of the larger circle is $\pi r^2 \text{ cm}^2$.

Since the eight congruent pieces each have the same area as the smaller circle, it follows that this area is equal to $\frac{1}{9}$ of the area of the larger circle. Thus,

$$\begin{aligned}25\pi &= \frac{1}{9}\pi r^2 \\25 &= \frac{1}{9}r^2 \\9 \times 25 &= r^2 \\225 &= r^2\end{aligned}$$

Thus, $r = 15$, since $r > 0$.

It follows that the circumference of the larger circle is $2\pi \times r = 2\pi \times 15 = 30\pi \text{ cm}$.