$$\begin{array}{r} \boxed{}\boxed{}\boxed{} \\ +\ \boxed{}\boxed{}\boxed{} \\ \hline 1\ 0\ 0\ 0 \end{array}$$

Problem of the Week

Problem C and Solution

Arranging Tiles 1

Problem

Ana has nine tiles, each with a different integer from 1 to 9 on it. Ana creates larger numbers by placing tiles side by side. For example, using the tiles 3 and 7, Ana can create the 2-digit number 37 or 73. Using six of her tiles, Ana forms two 3-digit numbers that add to 1000. What is the largest possible 3-digit number that she could have used?

Solution

We will use the letters A, B, C, D, E, and F to represent the integers on the six chosen tiles, letting the two 3-digit numbers be ABC and DEF. Then we will determine the largest possible 3-digit number ABC.

$$\begin{array}{r} \boxed{A}\boxed{B}\boxed{C} \\ +\ \boxed{D}\boxed{E}\boxed{F} \\ \hline 1\ 0\ 0\ 0 \end{array}$$

Looking at the ones column, since C and F are both digits from 1 to 9 and add to a number that ends in 0, their sum must be 10. (Their sum cannot be zero since neither C nor F is zero, and their sum cannot be 20 or more since C and F are each less than 10.) Thus, $C + F = 10$. Therefore, there is a carry of 1 into the tens column. Similarly, the sum in the tens column must also be 10, so $B + E + 1 = 10$, or $B + E = 9$. Therefore, there is a carry of 1 into the hundreds column. Thus, $A + D + 1 = 10$, or $A + D = 9$.

$$\begin{array}{r} {}^{1}\ \ {}^{1}\ \ \\ \boxed{A}\boxed{B}\boxed{C} \\ +\ \boxed{D}\boxed{E}\boxed{F} \\ \hline 1\ 0\ 0\ 0 \end{array}$$

To determine the largest possible 3-digit number ABC, A must be as large as possible. We have the following tiles: 1, 2, 3, 4, 5, 6, 7, 8, and 9. Since $A + D = 9$, A is largest when $A = 8$ and $D = 1$.

The next step is to make B as large as possible. We are left with the following tiles: 2, 3, 4, 5, 6, 7, and 9. Since $B + E = 9$, B is largest when $B = 7$ and $E = 2$.

Finally, we need to make C as large as possible. We are left with the following tiles: 3, 4, 5, 6, and 9. Since $C + F = 10$, then C is largest when $C = 6$ and $F = 4$.

Therefore, the largest possible 3-digit number ABC is 876.

Indeed, we can check that when ABC is 876, we have DEF equal to 124, and $ABC + DEF = 876 + 124 = 1000$.