Problem of the Week
Problem C and Solution
And the Numbers Are...

Problem

John and Betty each choose a positive integer that is greater than 1. Betty increases her number by 1. John then takes this new number and multiplies it by his number. This product is equal to 260.

If Betty's number is larger than John's number, determine all possible pairs of integers that John and Betty could have chosen.

Solution

Let John's integer be j and Betty's integer be b. We're given $j \times (b+1) = 260$.

In considering the equation $j \times (b+1) = 260$, we are looking for two integers, each greater than 1, that multiply to 260. We also want Betty's integer b to be greater than John's integer j.

We generate the following list of ways to factor 260 as a product of two integers:

$$1 \times 260, \ 2 \times 130, \ 4 \times 65, \ 5 \times 52, \ 10 \times 26, \ 13 \times 20$$

We can exclude $260 = 1 \times 260$ because both integers must be greater than 1.

Since Betty's integer is larger than John's integer, we get the following possibilities:

- $j = 2$ and $b + 1 = 130$. Thus, $j = 2$ and $b = 129$.

- $j = 4$ and $b + 1 = 65$. Thus, $j = 4$ and $b = 64$.

- $j = 5$ and $b + 1 = 52$. Thus, $j = 5$ and $b = 51$.

- $j = 10$ and $b + 1 = 26$. Thus, $j = 10$ and $b = 25$.

- $j = 13$ and $b + 1 = 20$. Thus, $j = 13$ and $b = 19$.

Therefore, there are five pairs of integers that John and Betty could have chosen. John could have chosen 2 and Betty chose 129, John could have chosen 4 and Betty chose 64, John could have chosen 5 and Betty chose 51, John could have chosen 10 and Betty chose 25, or John could have chosen 13 and Betty chose 19.