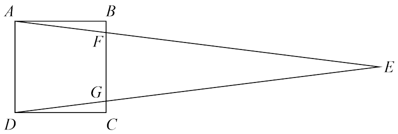


Problem of the Week
Problem D and Solution
Overlapping Shapes 2

Problem

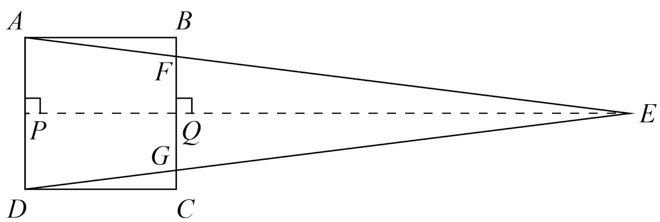
Selena draws square $ABCD$ with side length 16 cm. Endre then draws $\triangle AED$ on top of square $ABCD$ so that

- sides AE and DE meet BC at F and G, respectively, and
- the area of $\triangle AED$ is twice the area of square $ABCD$.

Determine the area of trapezoid $AFGD$.

Solution

We construct an altitude of $\triangle AED$ from E, intersecting AD at P and BC at Q. Since $ABCD$ is a square, we know that AD is parallel to BC. Therefore, since PE is perpendicular to AD, QE is perpendicular to FG and thus an altitude of $\triangle FEG$.



The area of square $ABCD$ is $16 \times 16 = 256$ cm^2. Since the area of $\triangle AED$ is twice the area of square $ABCD$, it follows that the area of $\triangle AED$ is $2 \times 256 = 512$ cm^2.

We also know that

$$\begin{aligned} \text{Area } \triangle AED &= AD \times PE \div 2 \\ 512 &= 16 \times PE \div 2 \\ 512 &= 8 \times PE \\ PE &= 512 \div 8 \\ &= 64 \text{ cm} \end{aligned}$$

Since $\angle APQ = 90^\circ$, we know that $ABQP$ is a rectangle, and so $PQ = AB = 16$ cm. We also know that $PE = PQ + QE$. Since $PE = 64$ cm and $PQ = 16$ cm, it follows that $QE = PE - PQ = 64 - 16 = 48$ cm.

From here we proceed with two different solutions.

Solution 1

We will use the relationships between the areas of the shapes to determine the length of FG.

$$\begin{aligned}
\text{Area of trapezoid } AFGD + \text{Area } \triangle FEG &= \text{Area } \triangle AED \\
(AD + FG) \times AB \div 2 + FG \times QE \div 2 &= 512 \\
(16 + FG) \times 16 \div 2 + FG \times 48 \div 2 &= 512 \\
(16 + FG) \times 8 + 24 \times FG &= 512 \\
128 + 8FG + 24FG &= 512 \\
32FG &= 384 \\
FG &= 12 \text{ cm}
\end{aligned}$$

Now we can use FG to calculate the area of trapezoid $AFGD$.

$$\begin{aligned}
\text{Area of trapezoid } AFGD &= (AD + FG) \times AB \div 2 \\
&= (16 + 12) \times 16 \div 2 \\
&= 28 \times 8 = 224 \text{ cm}^2
\end{aligned}$$

Therefore, the area of trapezoid $AFGD$ is 224 cm^2.

Solution 2

We will use similar triangles to determine the length of FG. We know that $\angle AED = \angle FEG$. Also, since AD is parallel to FG it follows that $\angle EAD$ and $\angle EFG$ are corresponding angles, so are equal. Thus, by angle-angle similarity, $\triangle AED \sim \triangle FEG$. Therefore,

$$\begin{aligned}
\frac{AD}{PE} &= \frac{FG}{QE} \\
\frac{16}{64} &= \frac{FG}{48} \\
\frac{1}{4} &= \frac{FG}{48} \\
FG &= 48 \times \frac{1}{4} = 12 \text{ cm}
\end{aligned}$$

Now we can use FG to calculate the area of trapezoid $AFGD$.

$$\begin{aligned}
\text{Area of trapezoid } AFGD &= (AD + FG) \times AB \div 2 \\
&= (16 + 12) \times 16 \div 2 \\
&= 28 \times 8 = 224 \text{ cm}^2
\end{aligned}$$

Therefore, the area of trapezoid $AFGD$ is 224 cm^2.