55555
4444
333
22

Problem of the Week
Problem D and Solution
Number Display

Problem

Helena's Hardware Store is clearing out a particular style of single digits that are used for house numbers. There are currently only five 5s, four 4s, three 3s, and two 2s left.

How many different three-digit house numbers can be made using these single digits?

Solution

Solution 1

Let's suppose that there were three or more 2s available. For the first digit, the customer could choose from the digits 5, 4, 3, and 2. Therefore, there would be 4 choices for the first digit. Similarly, there would be 4 choices for the second digit, and 4 choices for the third digit. This would give $4 \times 4 \times 4 = 64$ possible three-digit house numbers that could be made.

However, there are actually only two 2s available, so not all of these house numbers can be made. In particular, the house number 222 cannot be made, but all others can.

Therefore, $64 - 1 = 63$ different three-digit house numbers can be made using these single digits.

Solution 2

Let's look at three different cases.

Case 1: All three digits in the house number are the same
The house number could then be 555, 444, or 333. The number 222 cannot be made since only two 2s are available. Therefore, there are 3 three-digit house numbers with all three digits the same.

Case 2: Two digits are the same and the third digit is different
There are 4 choices for the digits that are the same, namely 5, 4, 3, and 2. For each of these possible choices, there are 3 choices for the third different digit. For example, if two of the digits are 5, then the third digit could be 4, 3, or 2. Therefore, there are $4 \times 3 = 12$ ways to choose the digits. For each of these choices, there are 3 ways to arrange the digits. For example, suppose the digits are a, a, and b. The house number could be aab, aba, or baa. Therefore, there are $12 \times 3 = 36$ three-digit house numbers with two digits the same and one different.

Case 3: All three digits are different
The customer has 4 choices for the first digit, namely 5, 4, 3, or 2. Once that digit is chosen, there are 3 choices for the second digit. Once the first and second digits are chosen, there are 2 choices for the third digit. Therefore, there are $4 \times 3 \times 2 = 24$ three-digit house numbers with all three digits different.

Therefore, $3 + 36 + 24 = 63$ different three-digit house numbers can be made using these single digits.