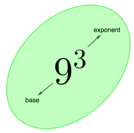

Problem of the Week

Problem D and Solution

The Same Power

Problem

Sometimes two powers that are not written with the same base are still equal in value. For example, $9^3 = 27^2$ and $(-5)^4 = 25^2$.

If x and y are integers, find all ordered pairs (x, y) that satisfy the equation

$$(x-1)^{x+y} = 8^2$$

Solution

Since $8^2 = 64$, we want to look at how we can express 64 as a^b where a and b are integers. There are six ways to do so. We can do so as 64^1, 8^2, 4^3, 2^6, $(-2)^6$, and $(-8)^2$.

We use these powers and the expression $(x-1)^{x+y}$ to find values for x and y.

- The power $(x-1)^{x+y}$ is expressed as 64^1 when $x - 1 = 64$ and $x + y = 1$. Then $x = 65$ and $y = -64$ follows. Thus $(65, -64)$ is one pair.
- The power $(x-1)^{x+y}$ is expressed as 8^2 when $x - 1 = 8$ and $x + y = 2$. Then $x = 9$ and $y = -7$ follows. Thus $(9, -7)$ is one pair.
- The power $(x-1)^{x+y}$ is expressed as 4^3 when $x - 1 = 4$ and $x + y = 3$. Then $x = 5$ and $y = -2$ follows. Thus $(5, -2)$ is one pair.
- The power $(x-1)^{x+y}$ is expressed as 2^6 when $x - 1 = 2$ and $x + y = 6$. Then $x = 3$ and $y = 3$ follows. Thus $(3, 3)$ is one pair.
- The power $(x-1)^{x+y}$ is expressed as $(-2)^6$ when $x - 1 = -2$ and $x + y = 6$. Then $x = -1$ and $y = 7$ follows. Thus $(-1, 7)$ is one pair.
- The power $(x-1)^{x+y}$ is expressed as $(-8)^2$ when $x - 1 = -8$ and $x + y = 2$. Then $x = -7$ and $y = 9$ follows. Thus $(-7, 9)$ is one pair.

Therefore, there are six ordered pairs that satisfy the equation.
They are $(65, -64)$, $(9, -7)$, $(5, -2)$, $(3, 3)$, $(-1, 7)$, and $(-7, 9)$.