



Problem of the Week

Problem D and Solution

Choices

Problem

Matilda and Yolanda each chose an integer. When Matilda's integer is multiplied by the sum of Matilda's and Yolanda's integers, the product is 299.

If Matilda's integer is smaller than Yolanda's integer, determine all possible pairs of integers that Matilda and Yolanda could have chosen.

Solution

Let x represent Matilda's integer and y represent the Yolanda's integer. We're given $x < y$.

The sum of the two integers is $x + y$. We multiply this sum by Matilda's integer, x . The resulting expression is $x(x + y)$. Thus, we want to find all pairs of integers satisfying $x(x + y) = 299$ with $x < y$.

We want the product of two integers to be 299. The factors of 299 are $\pm 1, \pm 13, \pm 23, \pm 299$. Thus, the possible values for x are $\pm 1, \pm 13, \pm 23, \pm 299$.

In the following table, we list all the possible values for x and then determine the corresponding value for y . If $x < y$, then this is a valid possibility. For example, if $x = 1$, then $x + y$ must be 299. Therefore, y must be 298. Since $x < y$, one possibility is Matilda chooses 1 and Yolanda chooses 298.

x	$x + y$	y	$x < y?$
1	299	298	Yes
13	23	10	No
23	13	-10	No
299	1	-298	No
-1	-299	-298	No
-13	-23	-10	Yes
-23	-13	10	Yes
-299	-1	298	Yes

Therefore, there are four pairs of integers that Matilda and Yolanda could have chosen. Matilda could have chosen 1 and Yolanda chose 298, Matilda could have chosen -13 and Yolanda chose -10, Matilda could have chosen -23 and Yolanda chose 10, or Matilda could have chosen -299 and Yolanda chose 298.