$n \overline{)2024}$ **?**

Problem of the Week
Problem D and Solution
Using Leftovers

Problem

A three-digit positive integer n has the property that when 2024 is divided by n, the remainder is 4. What is the sum of all such three-digit positive integers n?

Solution

Let p be the quotient when 2024 is divided by n. Since the remainder is 4, it follows that $np + 4 = 2024$. Thus, $np = 2020$.

Using the prime factorization of 2020 we obtain $2020 = 2 \times 2 \times 5 \times 101$. From this we can determine all the possible pairs of positive integers that multiply to 2020. These are summarized below.

$$1 \times 2020, \quad 2 \times 1010, \quad 4 \times 505, \quad 5 \times 404, \quad 10 \times 202, \quad 20 \times 101$$

Since n is a three-digit positive integer, it follows that the only possible values for n are 101, 202, 404, or 505. The sum of these is $101 + 202 + 404 + 505 = 1212$.