2023 Canadian Computing Olympiad
Day 1, Problem 1
Binaria

Time Limit: 1 second

Problem Description

You have been hired by the Cheap Communication Organization (CCO) to work on a communication breakthrough: sub-message sum (SMS). This revolutionary idea works as follows.

Given a binary string of length N, and some positive integer K with $K \leq N$, the SMS for the string consists of a sequence of $N - K + 1$ sums. The first sum in the sequence is the sum of digits 1 through K, the second sum is the sum of digits 2 through $K + 1$, and so on until the last sum which is the sum of digits $N - K + 1$ through N.

For example, if $K = 4$, the SMS of the binary string `110010` is 2,2,1. This is because $1 + 1 + 0 + 0 = 2$, $1 + 0 + 0 + 1 = 2$, and $0 + 0 + 1 + 0 = 1$.

Since you are a very junior developer, your job is not to find the original binary string from a given SMS, but rather the number of binary strings that could have formed this SMS.

Input Specification

The first line of input contains the two space-separated integers N and K where $1 \leq K \leq N$.

The second line of input contains $N - K + 1$ space-separated integers which is the SMS of at least one binary string.

Marks Awarded	Bounds on N	Additional Bounds on K
3 marks	$1 \leq N \leq 10$	$K \leq 3$
3 marks	$1 \leq N \leq 10$	None
4 marks	$1 \leq N \leq 1\,000$	$K \leq 10$
4 marks	$1 \leq N \leq 10^6$	$K \leq 20$
4 marks	$1 \leq N \leq 10^6$	$K \leq 3\,000$
7 marks	$1 \leq N \leq 10^6$	None

Output Specification

Output the remainder of T divided by the prime number $10^6 + 3$ where T is the positive integer equal to the total number of possible binary strings that correspond to the given SMS.

Sample Input

```
7 4
3 2 2 2
```

Output for Sample Input

```
3
```

Explanation of Output for Sample Input

The possible strings of length 7 are `1011001`, `1101010`, and `1110011`.

2023 Canadian Computing Olympiad
Day 1, Problem 2
Real Mountains

Time Limit: 5 seconds

Problem Description

Thanks to your help with cropping her picture, Rebecca's scenic photo is now featured on the front cover of the newest issue of her magazine. However, it seems that some of her readers still aren't pleased with the picture. In particular, they seem to believe that the mountain in the picture is fake!

For simplicity, we can describe the picture as a sequence of N columns of pixels. In the i-th column, the first h_i pixels from the bottom are of mountains. Her readers will only believe that the picture contains a real mountain if it contains a single (possibly wide) peak. That is, if there exists some index p with $1 \leq p \leq N$ such that $h_1 \leq h_2 \leq \cdots \leq h_p \geq \cdots \geq h_{N-1} \geq h_N$.

Luckily, Rebecca can still pay her editors to modify the picture and reprint the magazine. Unfortunately for her though, the editors have a very peculiar pricing scheme for their work. The only way Rebecca can edit the picture is by sending emails to her editors containing the integers (i, j, k) such that $1 \leq i < j < k \leq N$ and $h_i > h_j < h_k$. The editors will then add an extra pixel of mountains in the j-th column (i.e. increment h_j by 1) for a cost of $h_i + h_j + h_k$ cents. Note that the change in h_j may affect the costs of future edits.

To please her readers, Rebecca would like to edit the picture so that they believe it contains a real mountain. Can you tell her the minimum cost required to do so?

Input Specification

The first line of input contains an integer N.

The second line of input contains N space-separated integers $h_1, h_2, \ldots, h_N$.

Marks Awarded	**Bounds on N**	**Bounds and constraints on h_i**
3 marks	$3 \leq N \leq 5\,000$	$1 \leq h_i \leq 100$; $h_1 \geq h_2 \geq \cdots \geq h_p \leq \cdots \leq h_{N-1} \leq h_N$ for some p, $1 \leq p \leq N$
3 marks	$3 \leq N \leq 5\,000$	$1 \leq h_i \leq 100$
3 marks	$3 \leq N \leq 5\,000$	$1 \leq h_i \leq 10^6$
3 marks	$3 \leq N \leq 5\,000$	$1 \leq h_i \leq 10^9$
4 marks	$3 \leq N \leq 10^6$	$1 \leq h_i \leq 100$
5 marks	$3 \leq N \leq 10^6$	$1 \leq h_i \leq 10^6$
4 marks	$3 \leq N \leq 10^6$	$1 \leq h_i \leq 10^9$

Output Specification

Output the remainder of T divided by the prime number $10^6 + 3$ where T is the minimum cost (in cents) that Rebecca would need to incur in order to please her readers.

Sample Input

```
8
3 2 4 5 4 1 2 1
```

Output for Sample Input

```
14
```

Explanation of Output for Sample Input

Rebecca can send two emails, the first containing the integers $(2, 6, 7)$ and the second containing the integers $(1, 2, 5)$. The first email costs 5 cents and increases h_6 by 1, while the second email costs 9 cents and increases h_2 by 1.

The h_i values in the final picture will be $[3, 3, 4, 5, 4, 2, 2, 1]$. Her readers will believe this final picture contains a real mountain.

2023 Canadian Computing Olympiad
Day 1, Problem 3
Line Town

Time Limit: 2 seconds

Problem Description

The N residents of Line Town have arranged themselves in a line. Initially, the residents have happiness values of $h_1, h_2, \ldots, h_N$ from left to right along the line.

Since you are the mayor of Line Town, you are implementing the third pillar of your plan entitled "Community, Candy, and Organization" (CCO). As such, you have taken the mayoral power to swap the resident's locations. In one swap, you may tell two *adjacent* residents to swap their positions in the line. However, this swap will cause both residents to negate their happiness values.

You would like to perform some swaps so that the residents' happiness values are in non-decreasing order from left to right in the line. Determine whether this is possible, and if so, the minimum number of swaps needed.

Input Specification

The first line of input contains a single integer N.

The next line of input contains N integers $h_1, \ldots, h_N$ ($-10^9 \leq h_i \leq 10^9$), the happiness values of the residents from left to right.

Marks Awarded	**Bounds on N**	**Bounds on h_i**
3 marks	$1 \leq N \leq 2\,000$	$\lvert h_i\rvert = 1$ for all i
3 marks	$1 \leq N \leq 500\,000$	$\lvert h_i\rvert = 1$ for all i
3 marks	$1 \leq N \leq 2\,000$	$\lvert h_i\rvert \leq 1$ for all i
4 marks	$1 \leq N \leq 500\,000$	$\lvert h_i\rvert \leq 1$ for all i
4 marks	$1 \leq N \leq 2\,000$	$\lvert h_i\rvert \neq \lvert h_j\rvert$ for all $i \neq j$
3 marks	$1 \leq N \leq 500\,000$	$\lvert h_i\rvert \neq \lvert h_j\rvert$ for all $i \neq j$
2 marks	$1 \leq N \leq 2\,000$	No additional constraints.
3 marks	$1 \leq N \leq 500\,000$	No additional constraints.

Output Specification

On a single line, output the minimum number of swaps, or -1 if the task is impossible.

Sample Input 1

```
6
-2 7 -1 -8 2 8
```

Output for Sample Input 1

```
3
```

Explanation of Output for Sample Input 1

It is possible to perform 3 swaps as follows:

1. Swap the 2nd and 3rd resident so that the line becomes $[-2, 1, -7, -8, 2, 8]$.
2. Swap the 4th and 5th resident so that the line becomes $[-2, 1, -7, -2, 8, 8]$.
3. Swap the 3rd and 4th resident so that the line becomes $[-2, 1, 2, 7, 8, 8]$.

The residents are now arranged in non-decreasing order of happiness values as required. No non-decreasing arrangement can be obtained with less than 3 swaps.

Sample Input 2

```
4
1 -1 1 -1
```

Output for Sample Input 2

```
-1
```

Explanation of Output for Sample Input 2

There is no sequence of swaps that will place residents in non-decreasing order of happiness values.