Canadian
Mathematics
Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

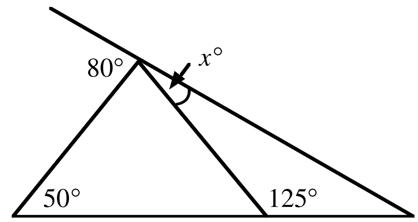
Fermat Contest *(Grade 11)*

Wednesday, February 19, 1997

Part A: Each question is worth 5 credits.

1. The value of $(1)^{10} + (-1)^8 + (-1)^7 + (1)^5$ is
(**A**) 0 (**B**) 4 (**C**) 1 (**D**) 16 (**E**) 2

2. The value of x is
(**A**) 15 (**B**) 20 (**C**) 25
(**D**) 30 (**E**) 35



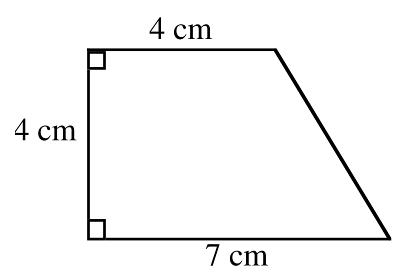
3. The greatest number of Mondays that can occur in 45 consecutive days is
(**A**) 5 (**B**) 6 (**C**) 7 (**D**) 8 (**E**) 9

4. The product of a positive number, its square, and its reciprocal is $\frac{100}{81}$. What is the number?
(**A**) $\frac{81}{100}$ (**B**) $\frac{100}{81}$ (**C**) $\frac{9}{10}$ (**D**) $\frac{10}{9}$ (**E**) $\frac{10\,000}{6561}$

5. The sum of seven consecutive positive integers is 77. The largest of these integers is
(**A**) 8 (**B**) 11 (**C**) 14 (**D**) 15 (**E**) 17

6. If 2×10^3 is represented as $2E3$ on a certain calculator, how would the product of $2E3$ and $3E2$ be represented?
(**A**) $6E6$ (**B**) $6E5$ (**C**) $5E5$ (**D**) $2.3E3$ (**E**) $5E6$

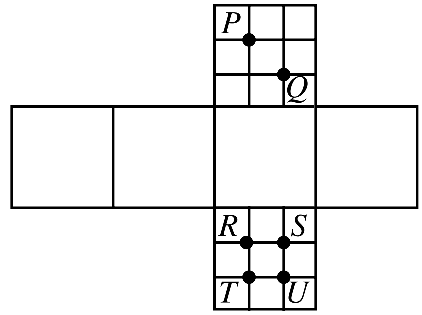
7. The perimeter of the figure shown is
(**A**) 19 cm (**B**) 22 cm (**C**) 21 cm
(**D**) 15 cm (**E**) 20 cm



8. Three of the vertices of a parallelogram are $(0,1)$, $(1,2)$, and $(2,1)$. The area of the parallelogram is
(**A**) 1 (**B**) 2 (**C**) $\sqrt{2}$ (**D**) $2\sqrt{2}$ (**E**) 4

9. If $10 \le x \le 20$ and $40 \le y \le 60$, the largest value of $\frac{x^2}{2y}$ is
(**A**) 5 (**B**) $\frac{5}{6}$ (**C**) $\frac{10}{3}$ (**D**) $\frac{5}{4}$ (**E**) 10

10. On a cube, two points are said to be diametrically opposite if the line containing the two points also contains the centre of the cube. The diagram below shows a pattern which could be folded into a cube. Which point would be diametrically opposite to point P?



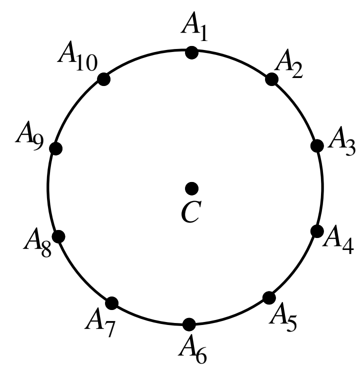
(**A**) Q (**B**) R (**C**) S (**D**) T (**E**) U

Part B: Each question is worth 6 credits.

11. Five integers have an average of 69. The middle integer (the median) is 83. The most frequently occurring integer (the mode) is 85. The range of the five integers is 70. What is the second smallest of the five integers?

(**A**) 77 (**B**) 15 (**C**) 50 (**D**) 55 (**E**) 49

12. On a circle, ten points $A_1, A_2, A_3, \ldots, A_{10}$ are equally spaced. If C is the centre of the circle, what is the size, in degrees, of the angle A_1A_5C?

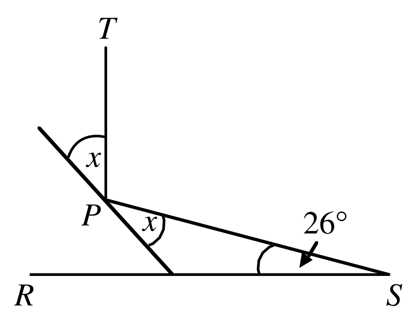


(**A**) 18 (**B**) 36 (**C**) 10 (**D**) 72 (**E**) 144

13. The digits 1, 2, 3, 4 can be arranged to form twenty-four different 4-digit numbers. If these twenty-four numbers are listed from smallest to largest, in what position is 3142?

(**A**) 13th (**B**) 14th (**C**) 15th (**D**) 16th (**E**) 17th

14. A beam of light shines from point S, reflects off a reflector at point P, and reaches point T so that PT is perpendicular to RS. Then x is



(**A**) 26° (**B**) 13° (**C**) 64° (**D**) 32° (**E**) 58°

15. If $x^2yz^3 = 7^3$ and $xy^2 = 7^9$, then xyz equals

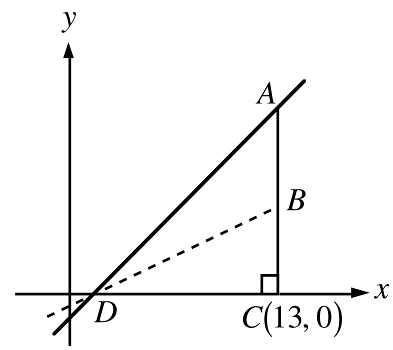
(**A**) 7^{10} (**B**) 7^9 (**C**) 7^8 (**D**) 7^6 (**E**) 7^4

16. The sum of the first 50 positive odd integers is 50^2. The sum of the first 50 positive even integers is
(**A**) 50^2 (**B**) 50^2+1 (**C**) 50^2+25 (**D**) 50^2+50 (**E**) 50^2+100

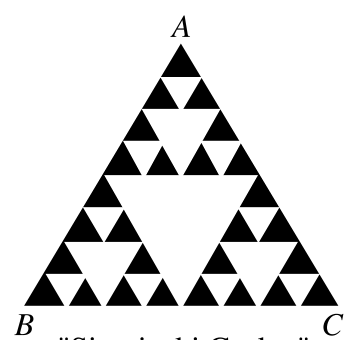
17. During 1996, the population of Sudbury decreased by 6% while the population of Victoria increased by 14%. At the end of the 1996, the populations of these cities were equal. What was the ratio of the population of Sudbury to the population of Victoria at the beginning of 1996?
(**A**) 47:57 (**B**) 57:47 (**C**) 53:43 (**D**) 53:57 (**E**) 43:47

18. Given $A=\{1,2,3,5,8,13,21,34,55\}$, how many of the numbers between 3 and 89 cannot be written as the sum of two elements of the set?
(**A**) 43 (**B**) 36 (**C**) 34 (**D**) 55 (**E**) 51

19. In the diagram, the equation of line AD is $y=\sqrt{3}(x-1)$. BD bisects $\angle ADC$. If the coordinates of B are (p,q), what is the value of q?
(**A**) 6 (**B**) 6.5 (**C**) $\frac{10}{\sqrt{3}}$
(**D**) $\frac{12}{\sqrt{3}}$ (**E**) $\frac{13}{\sqrt{3}}$



20. In the diagram, all triangles are equilateral. If $AB=16$, then the total area of all the black triangles is
(**A**) $37\sqrt{3}$ (**B**) $32\sqrt{3}$ (**C**) $27\sqrt{3}$
(**D**) $64\sqrt{3}$ (**E**) $\frac{64}{3}\sqrt{3}$



"Sierpinski Gasket"

Part C: Each question is worth 8 credits.

21. If $\dfrac{\left(\frac{a}{c}+\frac{a}{b}+1\right)}{\left(\frac{b}{a}+\frac{b}{c}+1\right)}=11$, and a, b, and c are positive integers, then the number of ordered triples (a,b,c), such that $a+2b+c\le 40$, is
(**A**) 33 (**B**) 37 (**C**) 40 (**D**) 42 (**E**) 45

22. If $2x^2 - 2xy + y^2 = 289$, where x and y are integers and $x \geq 0$, the number of different ordered pairs (x, y) which satisfy this equation is

(**A**) 8 (**B**) 7 (**C**) 5 (**D**) 4 (**E**) 3

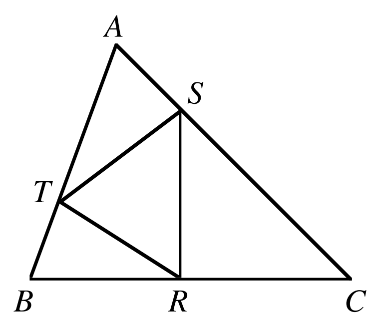
23. If $f(x) = px + q$ and $f(f(f(x))) = 8x + 21$, and if p and q are real numbers, then $p + q$ equals

(**A**) 2 (**B**) 3 (**C**) 5 (**D**) 7 (**E**) 11

24. The first term in a sequence of numbers is $t_1 = 5$. Succeeding terms are defined by the statement $t_n - t_{n-1} = 2n + 3$ for $n \geq 2$. The value of t_{50} is

(**A**) 2700 (**B**) 2702 (**C**) 2698 (**D**) 2704 (**E**) 2706

25. In triangle ABC, R is the mid-point of BC, $CS = 3SA$, and $\frac{AT}{TB} = \frac{p}{q}$. If w is the area of ΔCRS, x is the area of ΔRBT, z is the area of ΔATS, and $x^2 = wz$, then the value of $\frac{p}{q}$ is



(**A**) $\frac{\sqrt{21}-3}{2}$ (**B**) $\frac{\sqrt{21}+3}{2}$ (**C**) $\frac{\sqrt{21}-3}{6}$

(**D**) $\frac{\sqrt{105}+3}{6}$ (**E**) $\frac{\sqrt{105}-3}{6}$