Canadian
Mathematics
Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

1997 Solutions
Pascal Contest (Grade 9)

for the
NATIONAL BANK OF CANADA
Awards

PART A:

1. ***Solution***

$$\frac{4+35}{8-5} = \frac{39}{3}$$
$$= 13$$

ANSWER: (E)

2. ***Solution***

Since $268 + 189 = 457$, the ♥ represents the digit 5. ANSWER: (D)

3. ***Solution***

$$2\frac{1}{10} + 3\frac{11}{100} + 4\frac{111}{1000} = 2.1 + 3.11 + 4.111$$
$$= 9.321$$

ANSWER: (A)

4. ***Solution***

$$(1)^{10} + (-1)^8 + (-1)^7 + (1)^5 = 1 + 1 - 1 + 1$$
$$= 2$$

ANSWER: (C)

5. ***Solution 1***

Since 60% of the number equals 42, then 1% of the number equals $\frac{42}{60}$.

Therefore 50% of the number equals $50 \times \frac{42}{60}$ or 35. ANSWER: (D)

Solution 2

Let x represent the number.

Then $0.6x = 42$

$x = 70$

Thus, 50% of the number is $0.5(70) = 35$. ANSWER: (D)

6. ***Solution***

First, simplify the expression.

$$(x)\left(x^2\right)\left(\frac{1}{x}\right) = x^2$$

If $x = -2$, the value of the expression is $(-2)^2 = 4$. ANSWER: (A)

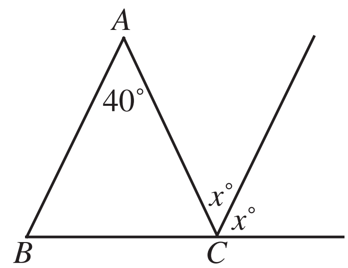
7. ***Solution***

Since ΔABC is isosceles, $\angle ABC = \angle ACB$.

The value of each of these angles is $\dfrac{(180° - 40°)}{2} = 70°$.

Since the points B, C, and D lie on a straight line,

$$70 + 2x = 180$$
$$2x = 110$$
$$x = 55$$



ANSWER: (B)

8. ***Solution***

If the first day is a Monday, then every seventh day is also a Monday, and Monday falls on the following numbered days: 1, 8, 15, 22, 29, 36, 43.

In the first 45 days of the year, the maximum number of Mondays is seven.

ANSWER: (C)

9. ***Solution***

The number is $9 \times 6 + 4 = 58$.

ANSWER: (A)

10. ***Solution***

Since the sum of the nine integers is 99, their average is $\frac{99}{9}$ or 11.

Because there is an odd number of consecutive integers, 11 is the middle integer and 15 is the largest.

ANSWER: (E)

PART B:

11. ***Solution***

The first ten balloons are popped in the following order: C, F, I, L, D, H, A, G, B, and K. The remaining two balloons are E and J.

ANSWER: (D)

12. ***Solution***

The total number of students answering the question was $300 + 1100 + 100 + 600 + 400 = 2500$.

Thus, the percentage of students who selected the correct response was $\left(\frac{1100}{2500}\right) \times 100$ or 44%.

ANSWER: (E)

13. ***Solution 1***

Since Janet has 10 coins, seven of which are dimes and quarters, then three coins are nickels.
Since eight coins are nickels or dimes, then five are dimes.
Thus Janet has 5 dimes. ANSWER: (D)

Solution 2
If Janet has n nickels, d dimes and q quarters, we can write

$$n+d+q=10 \qquad (1)$$

$$d+q=7 \qquad (2)$$

$$n+d=8 \qquad (3)$$

Subtracting equation (2) from equation (1), we get $n=3$. Substituting this in (3), we get $d=5$.
Thus, Janet has 5 dimes. ANSWER: (D)

14. ***Solution***
In the diagram, there are four small and two large triangles, for a total of 18 points. As well, there are four small and one large square, for a total of 20 points. Altogether, 38 points can be achieved. ANSWER: (A)

15. ***Solution***
The greatest possible value of p^q is 3^4 or 81.
The greatest possible value of p^q+r^s is 3^4+2^1 or 83. ANSWER: (D)

16. ***Solution***
In the diagram, there are 27 black triangles. If the entire diagram was divided into the smallest size equilateral triangles, there would be $8+2(7)+2(6)+2(5)+2(4)+2(3)+2(2)+2(1)=64$ (counting by rows). Thus, $\frac{27}{64}$ of ΔABC is coloured black. ANSWER: (E)

17. ***Solution***
The first twelve numbers in the list begin with either the digit 1 or 2. The next six begin with the digit 3. In order, these six numbers are 3124, 3142, 3214, 3241, 3412, 3421.
We see that the number 3142 is in the fourteenth position. ANSWER: (B)

18. ***Solution***

Since each factor of 10 produces a zero at the end of the integer we want to know how many 10's occur in the product.

The product of 20^{50} and 50^{20} can be rewritten as follows:

$$\begin{aligned}\left(20^{50}\right)\left(50^{20}\right) &= \left(2^2 \cdot 5\right)^{50}\left(5^2 \cdot 2\right)^{20} \\ &= 2^{100} \cdot 5^{50} \cdot 5^{40} \cdot 2^{20} \\ &= 2^{120} \cdot 5^{90} \\ &= 2^{30}\left(2^{90} \cdot 5^{90}\right) \\ &= 2^{30} \cdot 10^{90}\end{aligned}$$

From this, we see that there are 90 zeros at the end of the resulting integer.

ANSWER: (C)

19. ***Solution***

Each time a ball is drawn from the bag, there are three possible outcomes. Since a ball is drawn three different times, there are $3^3 = 27$ possible outcomes for the sum, although not all 27 are unique.

To determine how many of these outcomes give a sum that is less than eight, we first determine how many give a sum of eight or nine.

The only one way the sum of the three recorded numbers could be nine, is if three 3's are drawn.

To yield a sum of 8, the following three combinations are possible: 3, 3, 2 or 3, 2, 3 or 2, 3, 3.

Thus, of the 27 outcomes, $27 - 1 - 3 = 23$ give a sum less than 8, so the probability of obtaining the required sum is $\frac{23}{27}$.

ANSWER: (A)

20. ***Solution***

In the diagram, extend TP to meet RS at A. Since $AT \perp RS$, then $\angle SPA = 180° - 90° - 26°$

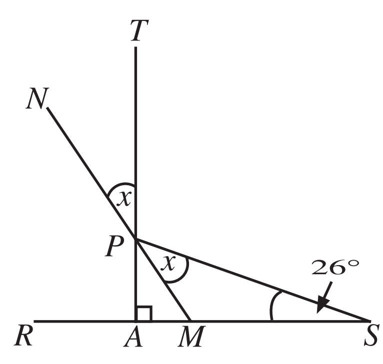
$= 64°$

Label points M and N. Since $\angle TPN$ and $\angle MPA$ are vertically opposite angles, they are equal, so $\angle MPA = x$.

Since $\angle SPA = 2x$, $\quad 2x = 64°$

$x = 32°$

Thus, the value of x is $32°$.

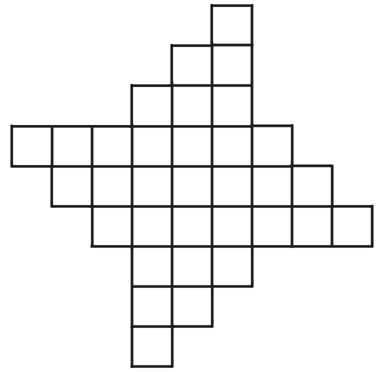


ANSWER: (B)

PART C:

21. ***Solution***

Since all of the shorter edges are equal in length, the diagram can be subdivided into 33 small squares, as shown. Each of these squares has area $\frac{528}{33} = 16$ and the length of each side is $\sqrt{16} = 4$.
By counting, we find 36 sides and a perimeter of 144.



ANSWER: (C)

22. ***Solution***

Rewrite $\frac{97}{19}$ as $5 + \frac{2}{19} = 5 + \frac{1}{\left(\frac{19}{2}\right)}$

$$= 5 + \frac{1}{9 + \frac{1}{2}}$$

By comparison, $w = 5$, $x = 9$ and $y = 2$.
Thus, $w + x + y = 16$. ANSWER: (A)

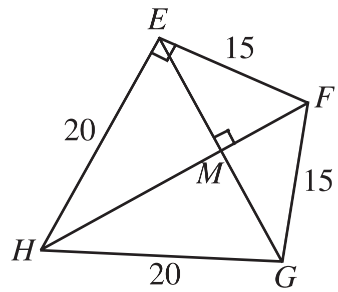
23. ***Solution***

Since 25 is the sum of two squares, the only possible values for x are 0, 3, 4 and 5. Substituting each value of x into the equation and finding the corresponding value of y gives five different values for y.

24. ***Solution***

Let the speed of the faster ship be x metres per second and the speed of the slower ship be y metres per second.
When the ships are travelling in opposite directions, their relative speed is $(x + y)$ m/s and the distance required to pass is 300 m, giving $10(x + y) = 300$.
When the ships are travelling in the same direction, their relative speed is $(x - y)$ m/s and the distance required to pass is still 300 m, giving $25(x - y) = 300$.
Solving for x gives $x = 21$.
Thus, the speed of the faster ship is 21 metres per second. ANSWER: (D)

25. ***Solution***



In order to calculate AG, we must first calculate EG. Consider the base quadrilateral $EFGH$ and join E to G and F to H. Where the diagonals meet, call the point M.

In ΔHGF apply Pythagorus to find $FH = 25$.

Notice that the triangles EFG and EHG are both isosceles so the diagonals of quadrilateral $EFGH$ meet at right angles.

ΔFMG and ΔFGH are similar triangles because they each contain a right angle and $\angle MFG$ is common to both. Using this similarity, $\frac{MG}{15} = \frac{20}{25}$, $MG = 12$ and thus, $EG = 24$.

Since $EG = 24$ and $AE = 32$, apply Pythagorus in ΔAEG to find $AG = 40$.

ANSWER: (B)