Canadian
Mathematics
Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

1999 Solutions

Gauss Contest

(Grades 7 and 8)

GRADE 7

Part A

1. $1999 - 999 + 99$ equals
   (**A**) 901 (**B**) 1099 (**C**) 1000 (**D**) 199 (**E**) 99

   Solution
   $1999 - 999 + 99$
   $= 1000 + 99$
   $= 1099$
   ANSWER: (B)

2. The integer 287 is exactly divisible by
   (**A**) 3 (**B**) 4 (**C**) 5 (**D**) 7 (**E**) 6

   Solution 1
   $\frac{287}{7} = 41$

   Solution 2
   If we think in terms of divisibility tests we see that:
   287 is not divisible by 3 because $2 + 8 + 7 = 17$ is not a multiple of 3;
   287 is not divisible by 4 because 87 is not divisible by 4;
   287 is not divisible by 5 because it doesn't end in 0 or 5;
   287 is divisible by 7 because $287 = 7 \times 41$;
   287 is not divisible by 6 because it is not even and is not divisible by 3.
   ANSWER: (D)

3. Susan wants to place 35.5 kg of sugar in small bags. If each bag holds 0.5 kg, how many bags are needed?
   (**A**) 36 (**B**) 18 (**C**) 53 (**D**) 70 (**E**) 71

   Solution
   Number of bags = $\frac{35.5}{.5} = \frac{355}{5} = 71$.
   ANSWER: (E)

4. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ is equal to
   (**A**) $\frac{15}{8}$ (**B**) $1\frac{3}{14}$ (**C**) $\frac{11}{8}$ (**D**) $1\frac{3}{4}$ (**E**) $\frac{7}{8}$

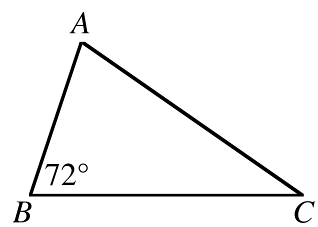
   Solution
   $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{8 + 4 + 2 + 1}{8} = \frac{15}{8}$
   ANSWER: (A)

5. Which one of the following gives an odd integer?
   (**A**) 6^2 (**B**) $23 - 17$ (**C**) 9×24 (**D**) $96 \div 8$ (**E**) 9×41

Solution 1
$6^2 = 36$, $23 - 17 = 6$, $9 \times 24 = 216$, $96 \div 8 = 12$, $9 \times 41 = 369$

Solution 2
If we think in terms of even and odd integers we have the following:
(A) (even)(even) = even
(B) odd – odd = even
(C) (odd)(even) = even
(D) (even) ÷ (even) = even or odd (It is necessary to evaluate)
(E) (odd)(odd) = odd ANSWER: (E)

6. In ΔABC, $\angle B = 72°$. What is the sum, in degrees, of the other two angles?
(**A**) 144 (**B**) 72 (**C**) 108
(**D**) 110 (**E**) 288



Solution
There are $180°$ in a triangle.
Therefore, $\angle A + \angle C + 72° = 180°$ ($\angle A$, $\angle B$ are in degrees.)
$\angle A + \angle C = 108°$. ANSWER: (C)

7. If the numbers $\frac{4}{5}$, 81% and 0.801 are arranged from smallest to largest, the correct order is
(**A**) $\frac{4}{5}$, 81%, 0.801 (**B**) 81%, 0.801, $\frac{4}{5}$ (**C**) 0.801, $\frac{4}{5}$, 81%

(**D**) 81%, $\frac{4}{5}$, 0.801 (**E**) $\frac{4}{5}$, 0.801, 81%

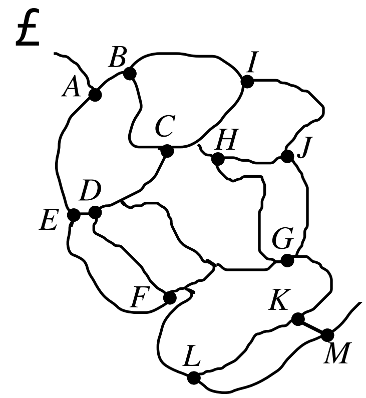
Solution
In decimal form, $\frac{4}{5} = .80$ and 81% = .81.
Arranging the given numbers from smallest to largest, we have $\frac{4}{5}$, 0.801, .81. ANSWER: (E)

8. The average of 10, 4, 8, 7, and 6 is
(**A**) 33 (**B**)13 (**C**) 35 (**D**) 10 (**E**) 7

Solution
$$\frac{10+4+8+7+6}{5} = \frac{35}{5} = 7$$
ANSWER: (E)

9. André is hiking on the paths shown in the map. He is planning to visit sites A to M in alphabetical order. He can never retrace his steps and he must proceed directly from one site to the next. What is the largest number of labelled points he can visit before going out of alphabetical order?

(**A**) 6 (**B**) 7 (**C**) 8
(**D**) 10 (**E**) 13

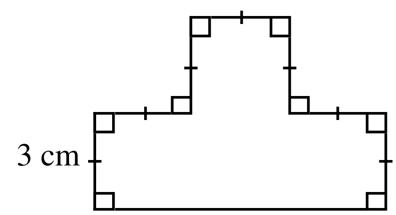


Solution
If we trace André's route we can see that he can travel to site J travelling in alphabetical order.
Once he reaches site J, it is not possible to reach K without passing through G or retracing his steps.
Since J is the tenth letter in the alphabet, he can visit ten sites before going out of order.

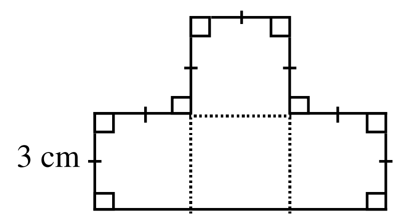
ANSWER: (D)

10. In the diagram, line segments meet at 90° as shown. If the short line segments are each 3 cm long, what is the area of the shape?

(**A**) 30 (**B**) 36 (**C**) 40
(**D**) 45 (**E**) 54



Solution
Each of the four squares are identical and each has an area of 3×3 or 9 cm^2.
The total area is thus 4×9 or 36 cm^2.



ANSWER: (B)

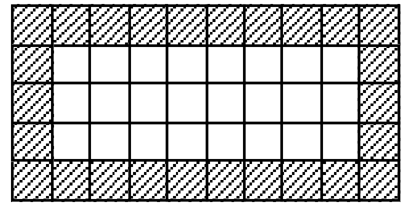
Part B

11. The floor of a rectangular room is covered with square tiles. The room is 10 tiles long and 5 tiles wide. The number of tiles that touch the walls of the room is

(**A**) 26 (**B**) 30 (**C**) 34 (**D**) 46 (**E**) 50

Solution

If we draw a grid that is 10×5, it is easy to count the number of tiles that touch the walls. From the diagram we can see that there are 26 tiles that touch the walls. Notice that if the question had said a length of l units (l an integer) and a width of w units (w an integer) we would arrive at the formula: $2w + 2l - 4$ where 4 represents the 4 corner tiles which would have been double counted. In this question, it would just be, $20 + 10 - 4 = 26$.



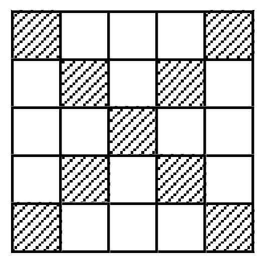
ANSWER: (A)

12. Five students named Fred, Gail, Henry, Iggy, and Joan are seated around a circular table in that order. To decide who goes first in a game, they play "countdown". Henry starts by saying '34', with Iggy saying '33'. If they continue to count down in their circular order, who will eventually say '1'?
(**A**) Fred (**B**) Gail (**C**) Henry (**D**) Iggy (**E**) Joan

Solution

This is an interesting question that mathematicians usually refer to as modular arithmetic. Henry starts by saying '34' and always says the number, $34 - 5n$, where n is a positive integer starting at 1. In other words, he says '34', '29', ..., '9', '4'. This implies Henry says '4', Iggy says '3', Joan says '2' and Fred says '1'. ANSWER: (A)

13. In the diagram, the percentage of small squares that are shaded is
(**A**) 9 (**B**) 33 (**C**) 36
(**D**) 56.25 (**E**) 64



Solution

There are 9 shaded squares out of a possible 25.
This represents, $\frac{9}{25}$ or 36%. ANSWER: (C)

14. Which of the following numbers is an odd integer, contains the digit 5, is divisible by 3, and lies between 12^2 and 13^2?
(**A**) 105 (**B**) 147 (**C**) 156 (**D**) 165 (**E**) 175

Solution

Since $12^2 = 144$ and $13^2 = 169$, we can immediately eliminate 105 and 175 as possibilities.
Since 156 is even it can also be eliminated. The only possibilities left are 147 and 165 but since 147 does not contain a 5 it can also be eliminated. The only candidate left is 165 and it can easily be checked that it meets the requirements of the question. ANSWER: (D)

15. A box contains 36 pink, 18 blue, 9 green, 6 red, and 3 purple cubes that are identical in size. If a cube

is selected at random, what is the probability that it is green?

(**A**) $\frac{1}{9}$ (**B**) $\frac{1}{8}$ (**C**) $\frac{1}{5}$ (**D**) $\frac{1}{4}$ (**E**) $\frac{9}{70}$

Solution

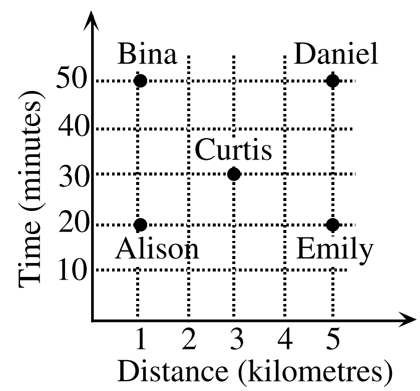
In total there are 72 cubes that have the same size.

Since there are 9 green cubes, the probability of selecting a green cube is $\frac{9}{72}$ or $\frac{1}{8}$.

ANSWER: (B)

16. The graph shown at the right indicates the time taken by five people to travel various distances. On average, which person travelled the fastest?

(**A**) Alison (**B**) Bina (**C**) Curtis
(**D**) Daniel (**E**) Emily



Solution

We summarize the results for the five people in the table.

We recall that average $\text{speed} = \frac{\text{distance}}{\text{time}}$.

	Distance	Time (minutes)	Speed (km/min.)
Alison	1	20	$\frac{1}{20} = 0.05$
Bina	1	50	$\frac{1}{50} = 0.02$
Curtis	3	30	$\frac{3}{30} = \frac{1}{10} = 0.1$
Daniel	5	50	$\frac{5}{50} = 0.1$
Emily	5	20	$\frac{5}{20} = 0.25$

Emily is the fastest.

ANSWER: (E)

17. In a "Fibonacci" sequence of numbers, each term beginning with the third, is the sum of the previous two terms. The first number in such a sequence is 2 and the third is 9. What is the eighth term in the sequence?

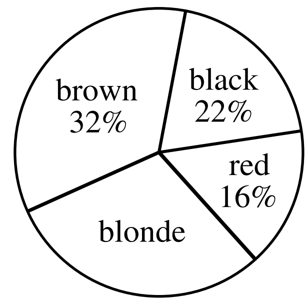
(**A**) 34 (**B**) 36 (**C**) 107 (**D**) 152 (**E**) 245

Solution
If the first number in the sequence is 2 and the third is 9, the second number in the sequence must be 7. The sequence is thus: 2, 7, 9, 16, 25, 41, 66, 107. The eighth term is 107.

ANSWER: (C)

18. The results of a survey of the hair colour of 600 people are shown in this circle graph. How many people have blonde hair?

(**A**) 30 (**B**) 160 (**C**) 180
(**D**) 200 (**E**) 420



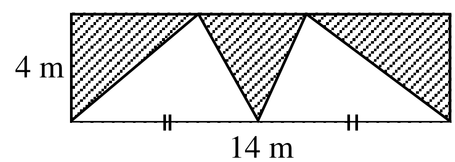
Hair Colour

Solution
From the diagram, blondes represent 30% of the 600 people.
Since 30% of $600 = (.3)(600) = 180$, there are 180 blondes in the survey.

ANSWER: (C)

19. What is the area, in m^2, of the shaded part of the rectangle?

(**A**) 14 (**B**) 28 (**C**) 33.6
(**D**) 56 (**E**) 42



Solution
The two unshaded triangles each have a base of 7 m and a height of 4 m. This means that each of the triangles has an area of $\frac{7 \times 4}{2} = 14\ \text{m}^2$. The two triangles thus have a total area of $28\ \text{m}^2$.
The shaded triangles have an area of $56 - 28 = 28\ \text{m}^2$.

ANSWER: (B)

20. The first 9 positive odd integers are placed in the magic square so that the sum of the numbers in each row, column and diagonal are equal. Find the value of $A + E$.

(**A**) 32 (**B**) 28 (**C**) 26
(**D**) 24 (**E**) 16

A	1	B
5	C	13
D	E	3

Solution
The first nine odd positive integers sum to 81.
This implies that the sum of each column is $\frac{81}{3}$ or 27. From this we immediately see that $B = 11$ since $B + 13 + 3 = 27$. If we continue with the constraint that each row or column must add to 27 then $A = 15 \rightarrow D = 7 \rightarrow E = 17$. Therefore, $A + E = 15 + 17 = 32$.

ANSWER: (A)

Part C

21. A game is played on the board shown. In this game, a player can move three places in any direction (up, down, right or left) and then can move two places in a direction perpendicular to the first move. If a player starts at S, which position on the board (P, Q, R, T, or W) cannot be reached through any sequence of moves?

		P		
	Q		R	
		T		
S				W

(A) P **(B)** Q **(C)** R
(D) T **(E)** W

Solution
If S is the starting position we can reach position R immediately. From S we can also reach P and then W and Q in sequence. To reach position T, it would have to be reached from the upper right or upper left square. There is no way for us to reach these two squares unless we are allowed to move outside the large square which is not permitted.

ANSWER: (D)

22. Forty-two cubes with 1 cm edges are glued together to form a solid rectangular block. If the perimeter of the base of the block is 18 cm, then the height, in cm, is

(A) 1 **(B)** 2 **(C)** $\frac{7}{3}$ **(D)** 3 **(E)** 4

Solution 1
Since we have a solid rectangular block with a volume of 42, its dimensions could be, $42 \times 1 \times 1$ or $6 \times 7 \times 1$ or $21 \times 2 \times 1$ or $2 \times 3 \times 7$ or $14 \times 3 \times 1$.
The only selection which has two factors adding to 9 is $2 \times 3 \times 7$, thus giving the base a perimeter of $2(2+7) = 18$ which is required.
So the base is 2×7 and the height is 3.

Solution 2
Since the perimeter of the base is 18 cm, the length and width can only be one of the following:

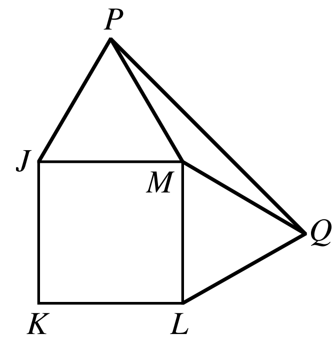
L	W
8	1
7	2
6	3
5	4

If the height is h, these cases lead to the following:
$8 \times 1 \times h = 42$, $7 \times 2 \times h = 42$, $6 \times 3 \times h = 42$ or $5 \times 4 \times h = 42$.
The only possible value of h which is an integer is $h = 3$, with $L = 7$ and $W = 2$.

ANSWER: (D)

23. $JKLM$ is a square. Points P and Q are outside the square such that triangles JMP and MLQ are both equilateral. The size, in degrees, of angle PQM is

(**A**) 10 (**B**) 15 (**C**) 25
(**D**) 30 (**E**) 150



Solution

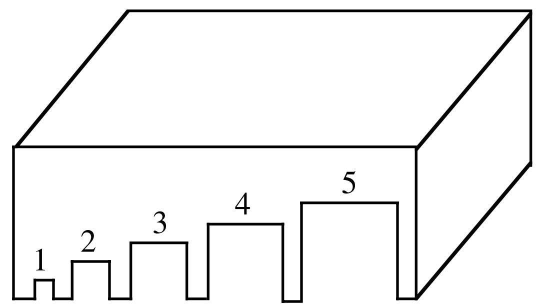
$\angle PMQ = 360° - 90° - 60° - 60° = 150°$

Since ΔPQM is isosceles, $\angle PQM = \frac{180° - 150°}{2} = 15°$.

ANSWER: (B)

24. Five holes of increasing size are cut along the edge of one face of a box as shown. The number of points scored when a marble is rolled through that hole is the number above the hole. There are three sizes of marbles: small, medium and large. The small marbles fit through any of the holes, the medium fit only through holes 3, 4 and 5 and the large fit only through hole 5. You may choose up to 10 marbles of each size to roll and every rolled marble goes through a hole. For a score of 23, what is the maximum number of marbles that could have been rolled?

(**A**) 12 (**B**) 13 (**C**) 14
(**D**) 15 (**E**) 16



Solution

We are looking for a ***maximum*** so we want to use lots of marbles. Let's start with 10 small ones. If they all go through hole #1, we have $23 - 10 = 13$ points to be divided between medium and large marbles. We could use 2 large and 1 medium $(5 + 5 + 3 = 13)$ and thus use $10 + 3 = 13$ marbles or we could use 4 medium and have one of these go through hole #4 $(3 + 3 + 3 + 4 = 13)$ which gives 14 marbles. Alternatively, of the 10 small marbles, if 9 go through hole #1 and 1 goes through hole #2, we have scored 11 points. The 4 medium marbles can now go through hole #3 giving a score of $11 + 3 \times 4 = 23$. This again gives a total of 14 marbles.

ANSWER: (C)

25. In a softball league, after each team has played every other team 4 times, the total accumulated points are: Lions 22, Tigers 19, Mounties 14, and Royals 12. If each team received 3 points for a win, 1 point for a tie and no points for a loss, how many games ended in a tie?

(**A**) 3 (**B**) 4 (**C**) 5 (**D**) 7 (**E**) 10

Solution

When every team plays every other team there are $3+2+1=6$ games. Since each team plays each of the other teams 4 times, there are $4(6)=24$ games.

When there is a winner 3 points are awarded. If each of the 24 games had winners there would be $24\times 3=72$ points awarded. The actual point total is $22+19+14+12=67$.

When there are ties, only $1+1=2$ points are awarded and so every point below 72 represents a tie.

Thus, the number of ties is $72-67=5$. ANSWER: (C)

GRADE 8

Part A

1. $10^3 + 10^2 + 10$ equals

(**A**) 1110 (**B**) 101 010 (**C**) 111 (**D**) 100 010 010 (**E**) 11 010

Solution

$10^3 + 10^2 + 10$

$= 1000 + 100 + 10$

$= 1110$

ANSWER: (A)

2. $\frac{1}{2} + \frac{1}{3}$ is equal to

(**A**) $\frac{2}{5}$ (**B**) $\frac{1}{6}$ (**C**) $\frac{1}{5}$ (**D**) $\frac{3}{2}$ (**E**) $\frac{5}{6}$

Solution 1

$\frac{1}{2} + \frac{1}{3}$

$= \frac{3}{6} + \frac{2}{6}$

$= \frac{5}{6}$

ANSWER: (E)

3. Which one of the following gives an odd integer?

(**A**) 6^2 (**B**) $23 - 17$ (**C**) 9×24 (**D**) 9×41 (**E**) $96 \div 8$

Solution 1

We can calculate each of the answers directly.

(**A**) $6^2 = 36$ (**B**) $23 - 17 = 6$ (**C**) $9 \times 24 = 216$ (**D**) $9 \times 41 = 369$ (**E**) $96 \div 8 = 12$

Solution 2

If we think in terms of even and odd integers we have the following:

(**A**) (even)(even) = even

(**B**) odd – odd = even

(**C**) (odd)(even) = even

(**D**) (odd)(odd) = odd

(**E**) (even) ÷ (even) = even or odd - the result must be calculated.

ANSWER: (D)

4. What is the remainder when 82 460 is divided by 8?

(**A**) 0 (**B**) 5 (**C**) 4 (**D**) 7 (**E**) 2

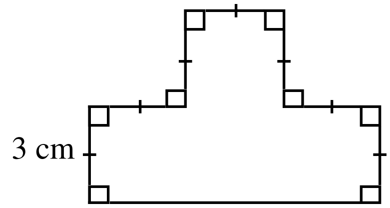
Solution

When considering division by 8, it is only necessary to consider division using the last 3 digits.
In essence, then, we are asking for the remainder when 460 is divided by 8.

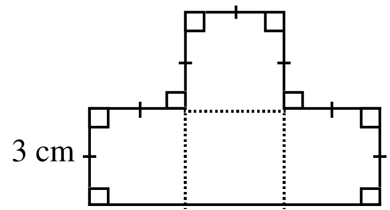
Since $460 = 8 \times 57 + 4$, the remainder is 4. ANSWER: (C)

5. In the diagram, line segments meet at 90° as shown. If the short line segments are each 3 cm long, what is the area of the shape?

(**A**) 30 (**B**) 36 (**C**) 40
(**D**) 45 (**E**) 54



Solution
Each of the four squares are identical and each has an area of 3×3 or $9\ \text{cm}^2$.
The total area is thus 4×9 or $36\ \text{cm}^2$.



ANSWER: (B)

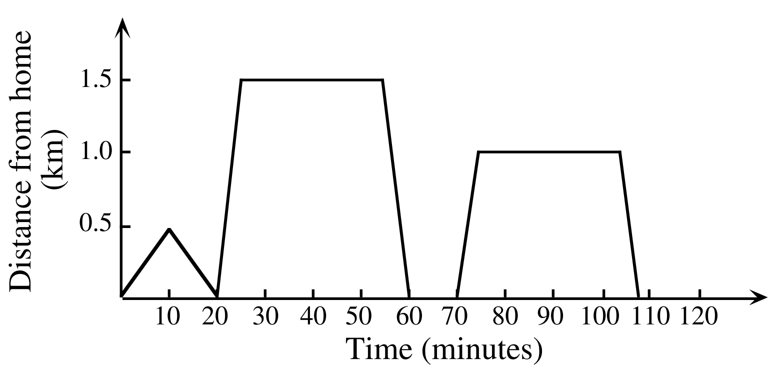
6. The average of -5, -2, 0, 4, and 8 is
(**A**) 1 (**B**) 0 (**C**) $\frac{19}{5}$ (**D**) $\frac{5}{4}$ (**E**) $\frac{9}{4}$

Solution
The average is, $\dfrac{(-5)+(-2)+(0)+(4)+(8)}{5} = 1.$ ANSWER: (A)

7. If the sales tax rate were to increase from 7% to 7.5%, then the tax on a $1000 item would go up by
(**A**) $75.00 (**B**) $5.00 (**C**) $0.5 (**D**) $0.05 (**E**) $7.50

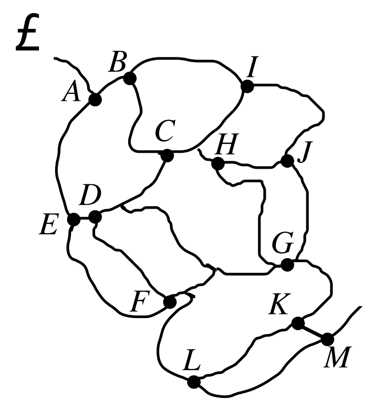
Solution
If the sales tax rate increases by .5%, this would represent an increase of $.50 on each $100.
Thus the increase in tax would be $(10)(\$.50) = \$5.00$. ANSWER: (B)

8. Tom spent part of his morning visiting and playing with friends. The graph shows his travels. He went to his friends' houses and stopped to play if they were at home. The number of houses at which he stopped to play is
(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5



Solution
From the graph, we can see that Tom stops at two houses in his travels. Notice that his first visit to a house, illustrated by the 'triangular' shape implies that his friend was not at home.
During his other two visits, the horizontal line indicates the fact that Tom stayed.
In both instances, Tom stayed for about 30 minutes. ANSWER: (B)

9. André is hiking on the paths shown in the map. He is planning to visit sites A to M in alphabetical order. He can never retrace his steps and he must proceed directly from one site to the next. What is the largest number of labelled points he can visit before going out of alphabetical order?
(**A**) 6 (**B**) 7 (**C**) 8
(**D**) 10 (**E**) 13



Solution
If we trace André's route we can see that he can travel to site J travelling in alphabetical order.
Once he reaches site J, it is not possible to reach K without passing through G or retracing his steps.
Since J is the tenth letter in the alphabet, he can visit ten sites before going out of order.
ANSWER: (D)

10. The area of a rectangular shaped garden is 28 m^2. It has a length of 7 m. Its perimeter, in metres, is
(**A**) 22 (**B**) 11 (**C**) 24 (**D**) 36 (**E**) 48

Solution
If the garden has a length of 7 m then its width will be 4 m.
Its perimeter is $2(4+7)=22$ m. ANSWER: (A)

Part B

11. Which of the following numbers is an odd integer, contains the digit 5, is divisible by 3, and lies between 12^2 and 13^2?
(**A**) 105 (**B**) 147 (**C**) 156 (**D**) 165 (**E**) 175

Solution
Since $12^2=144$ and $13^2=169$, we can immediately eliminate 105 and 175 as possibilities.
Since 156 is even it can also be eliminated. The only possibilities left are 147 and 165 but since 147 does not contain a 5 it can also be eliminated. The only candidate left is 165 and it can easily be checked that it meets the requirements of the question. ANSWER: (D)

12. If $\frac{n+1999}{2}=-1$, then the value of n is

(**A**) -2001 (**B**) -2000 (**C**) -1999 (**D**) -1997 (**E**) 1999

Solution

By inspection or by multiplying each side by 2, we arrive at $n+1999=-2$ or $n=-2001$.

ANSWER: (A)

13. The expression $n!$ means the product of the positive integers from 1 to n. For example, $5!=1\times2\times3\times4\times5$. The value of $6!-4!$ is

(**A**) 2 (**B**) 18 (**C**) 30 (**D**) 716 (**E**) 696

Solution

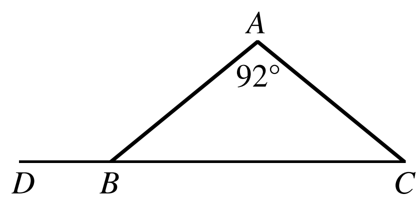
According to the given definition, $6!=6\times5\times4\times3\times2\times1=720$.

Similarly, $4!=24$. As a result, $6!-4!=720-24=696$.

ANSWER: (E)

14. ABC is an isosceles triangle in which $\angle A=92°$. CB is extended to a point D. What is the size of $\angle ABD$?

(**A**) 88° (**B**) 44° (**C**) 92°
(**D**) 136° (**E**) 158°



Solution

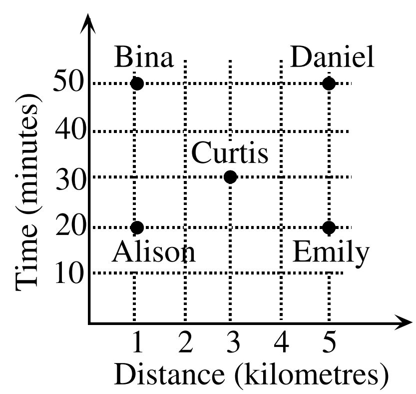
Since $\angle A=92°$ then $\angle ABC=\angle ACB=\frac{180°-92°}{2}=44°$.

Therefore, $\angle ABD=180°-44°=136°$.

ANSWER: (D)

15. The graph shown at the right indicates the time taken by five people to travel various distances. On average, which person travelled the fastest?

(**A**) Alison (**B**) Bina (**C**) Curtis
(**D**) Daniel (**E**) Emily



Solution

We summarize the results for the five people in the table.

We recall that average speed $=\frac{\text{distance}}{\text{time}}$.

	Distance	Time (minutes)	Speed (km/min.)
Alison	1	20	$\frac{1}{20} = 0.05$
Bina	1	50	$\frac{1}{50} = 0.02$
Curtis	3	30	$\frac{3}{30} = \frac{1}{10} = 0.1$
Daniel	5	50	$\frac{5}{50} = 0.1$
Emily	5	20	$\frac{5}{20} = 0.25$

Emily is the fastest.

ANSWER: (E)

16. In a set of five numbers, the average of two of the numbers is 12 and the average of the other three numbers is 7. The average of all five numbers is

(**A**) $8\frac{1}{3}$ (**B**) $8\frac{1}{2}$ (**C**) 9 (**D**) $8\frac{3}{4}$ (**E**) $9\frac{1}{2}$

Solution

In order that two numbers have an average of 12, the sum of the two numbers must have been 24.
Similarly, the three numbers must have had a sum of 21.
Thus the average of the five numbers is, $\frac{21+24}{5} = 9$.

ANSWER: (C)

17. In the subtraction question, $\begin{array}{r} 1957 \\ a9 \\ \hline 18b8 \end{array}$, the sum of the digits a and b is

(**A**) 15 (**B**) 14 (**C**) 10 (**D**) 5 (**E**) 4

Solution 1

If we treat the question as an ordinary subtraction question we get the following:

$$\begin{array}{cccc} & 8 & 14 & \\ 1 & \not{9} & \not{5} & 17 \\ & & a & 9 \\ \hline 1 & 8 & b & 8 \end{array}$$

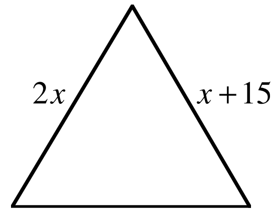
From this, $14 - a = b$ or $a + b = 14$.

Solution 2

Using trial and error, we can try different possibilities for a and b.
A good starting point is the possibility that $a + b = 15$ and $a = 8$, $b = 7$, for example. If we simply do the arithmetic this does not work. Of (**A**), (**B**), and (**C**), the only one that works is $a + b = 14$.
From observation, it should be clear that if $a + b = 4$ or 5 that the digits a and b would be far too small for the subtraction to work.

ANSWER: (B)

18. The equilateral triangle has sides of $2x$ and $x+15$ as shown. The perimeter of the triangle is



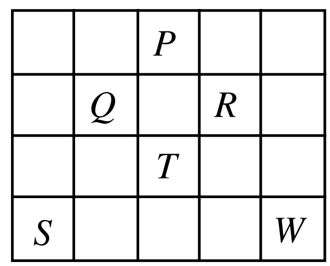
(**A**) 15 (**B**) 30 (**C**) 90
(**D**) 45 (**E**) 60

Solution
Since we are told the triangle is equilateral, $2x = x + 15$, or $x = 15$.
This makes the side length 30 and the perimeter 90. ANSWER: (C)

19. In a traffic study, a survey of 50 moving cars is done and it is found that 20% of these contain more than one person. Of the cars containing only one person, 60% of these are driven by women. Of the cars containing just one person, how many were driven by men?
(**A**) 10 (**B**) 16 (**C**) 20 (**D**) 30 (**E**) 40

Solution
The number of cars containing one person is 80% of 50, which is 40. Since 40% of these 40 cars are driven by men, the number driven by men is $.4 \times 40$ or 16. ANSWER: (B)

20. A game is played on the board shown. In this game, a player can move three places in any direction (up, down, right or left) and then can move two places in a direction perpendicular to the first move. If a player starts at *S*, which position on the board (*P*, *Q*, *R*, *T*, or *W*) cannot be reached through any sequence of moves?



(**A**) *P* (**B**) *Q* (**C**) *R*
(**D**) *T* (**E**) *W*

Solution
If *S* is the starting position we can reach position *R* immediately. From *S* we can also reach *P* and then *W* and *Q* in sequence. To reach position *T*, it would have to be reached from the upper right or upper left square. There is no way for us to reach these two squares unless we are allowed to move outside the large square which is not permitted.

ANSWER: (D)

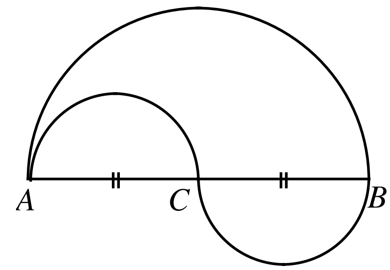
Part C

21. The sum of seven consecutive positive integers is always
(**A**) odd (**B**) a multiple of 7 (**C**) even
(**D**) a multiple of 4 (**E**) a multiple of 3

Solution
The easiest way to do this is to start with $1+2+3+4+5+6+7=28$. If we consider the next possibility, $2+3+4+5+6+7+8=35$, we notice that the acceptable sums are one of,

$\{28, 35, 42, 49, ...\}$. Each one of these numbers is a multiple of 7. ANSWER: (B)

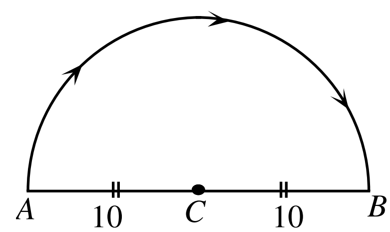
22. In the diagram, $AC = CB = 10$ m, where AC and CB are each the diameter of the small equal semi-circles. The diameter of the larger semi-circle is AB. In travelling from A to B, it is possible to take one of two paths. One path goes along the semi-circular arc from A to B. A second path goes along the semi-circular arcs from A to C and then along the semi-circular arc from C to B. The difference in the lengths of these two paths is



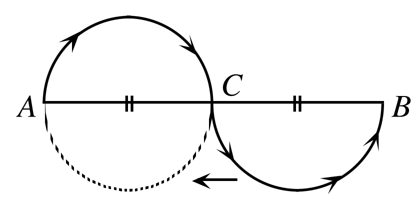
(**A**) 12π (**B**) 6π (**C**) 3π
(**D**) 2π (**E**) 0

Solution
Consider the two calculations.

Calculation 1
The distance travelled here would be one-half the circumference of the circle with radius 10.
This distance would be $\frac{1}{2}[2\pi(10)] = 10\pi$.



Calculation 2
The distance travelled would be the equivalent to the circumference of a circle with radius 5. The distance would be $2\pi(5) = 10\pi$.
Since these distances are equal, their difference would be $10\pi - 10\pi = 0$.



ANSWER: (E)

23. Kalyn writes down all of the integers from 1 to 1000 that have 4 as the sum of their digits. If $\frac{a}{b}$ (in lowest terms) is the fraction of these numbers that are prime, then $a + b$ is
(**A**) 5 (**B**) 4 (**C**) 15 (**D**) 26 (**E**) 19

Solution
The numbers between 1 and 1000 that have 4 as the sum of their digits are 4, (13), 22, (31), 40, (103), 112, 121, 130, 202, (211), 220, 301, 310, 400.
The circled numbers are prime which means that 4 out of the 15 are prime and $a + b = 19$.

ANSWER: (E)

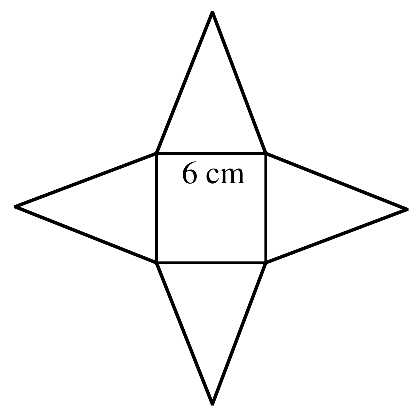
24. Raymonde's financial institution publishes a list of service charges as shown in the table. For her first twenty five transactions, she uses Autodebit three times as often as she writes cheques. She also writes as many cheques as she makes cash withdrawals. After her twenty-fifth transaction, she begins to make single transactions. What is the smallest number of transactions she needs to make so that her monthly service charges will exceed the $15.95 'all-in-one' fee?

	Service Fee per Item
Cheque	$0.50
Autodebit	$0.60
Cash Withdrawal	$0.45
'All-in-one' fee is $15.95	

(**A**) 29 (**B**) 30 (**C**) 27
(**D**) 28 (**E**) 31

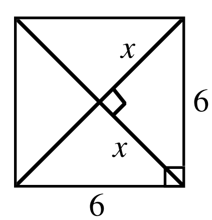
Solution
For Raymonde's first twenty five transactions, each set of five would cost $.50+.45+3(.60)=2.75$. After 25 transactions, her total cost would be $13.75. In order to exceed $15.95, she would have to spend $2.20. In order to minimize the number of transactions, she would use Autodebit four times. In total, the number of transactions would be $25+4=29$. ANSWER: (A)

25. Four identical isosceles triangles border a square of side 6 cm, as shown. When the four triangles are folded up they meet at a point to form a pyramid with a square base. If the height of this pyramid is 4 cm, the total area of the four triangles and the square is



(**A**) 84 cm^2 (**B**) 98 cm^2 (**C**) 96 cm^2
(**D**) 108 cm^2 (**E**) 90 cm^2

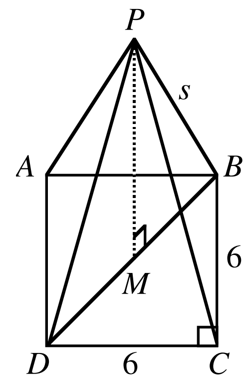
Solution
We draw in the two diagonals of the base square and label as shown. We can now say,



$$x^2+x^2=36$$
$$2x^2=36$$
$$x^2=18$$
$$x=\sqrt{18}.$$

Note: This question would work out very nicely if we had used $\sqrt{18} \doteq 4.24$ instead of the exact form, i.e. $\sqrt{18}$.

In this part of the solution, we have drawn the completed pyramid and labelled it as shown. We draw a line perpendicular to the square base from *P*. By using the fact that the pyramid has a square base and its sides are equal we conclude that this perpendicular line will pass through the mid-point of diagonal DB at the point *M*.



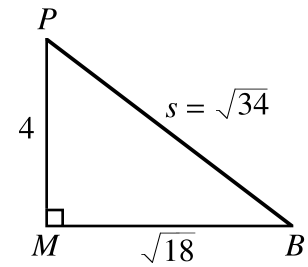
Using ΔPMB, we can now calculate the side length, *s*, of the pyramid.

$s^2 = 4^2 + \left(\sqrt{18}\right)^2$; Note that $x = MB = \sqrt{18}$

$s^2 = 16 + 18$

$s^2 = 34$

Therefore $s = \sqrt{34}$.



If we wish to calculate the height of the side triangles, which are each isosceles, we once again draw a perpendicular from *P* to the mid-point of one side of the square. We use ΔPAB and label the mid-point of AB point *N*. (Since ΔPAB is isosceles, the point *N* is the mid-point of AB.) Once again, we use pythagoras to calculate the heights of the isosceles triangles.

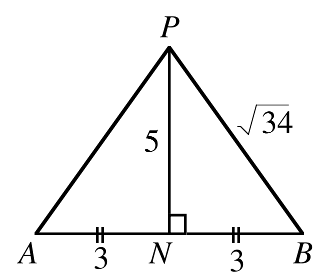
$$PB^2 = PN^2 + NB^2$$

$$\left(\sqrt{34}\right)^2 = PN^2 + 3^2$$

$$PN^2 = 34 - 9$$

$$PN^2 = 25$$

$$PN = 5.$$



We thus conclude that the height of each triangle is 5 and the area of each side triangle is $\frac{6 \times 5}{2} = 15$.

Thus, the total area is $4 \times 15 + 6 \times 6 = 96$. ANSWER: (C)