



Canadian Mathematics Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 23, 2000

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Time: 1 hour

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Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 20.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

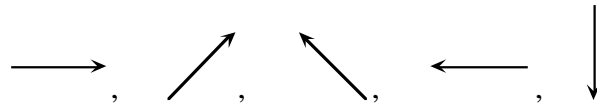
Scoring: There is *no penalty* for an incorrect answer.
 Each unanswered question is worth 2 credits, to a maximum of 20 credits.

Part A: Each correct answer is worth 5.

1. The value of $2(5-2)-5^2$ is

(A) -19 (B) -4 (C) 1 (D) -11 (E) -17

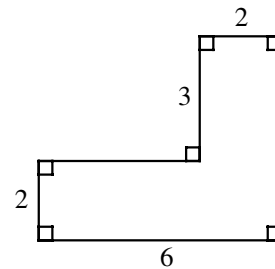
2. If the following sequence of five arrows repeats itself continuously, what arrow would be in the 48th position?



(A) (B) (C) (D) (E)

3. In the given diagram, the numbers shown are the lengths of the sides. What is the perimeter of the figure?

(A) 13 (B) 18 (C) 22
 (D) 21 (E) 19

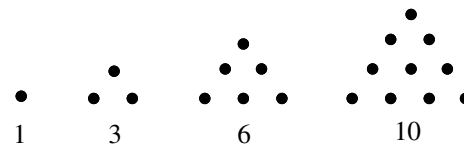


4. A farmer has 7 cows, 8 sheep and 6 goats. How many more goats should be bought so that half of her animals will be goats?

(A) 18 (B) 15 (C) 21 (D) 9 (E) 6

5. The first four triangular numbers 1, 3, 6 and 10 are illustrated in the diagram. What is the tenth triangular number?

(A) 55 (B) 45 (C) 66
 (D) 78 (E) 50

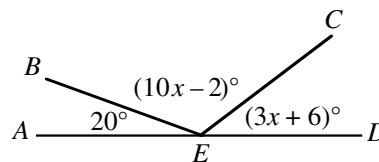


6. The sum of the digits of an even ten digit integer is 89. The last digit is

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

7. If AD is a straight line segment and E is a point on AD , determine the measure of $\angle CED$.

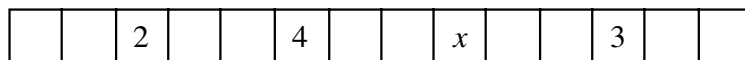
(A) 20° (B) 12° (C) 42°
 (D) 30° (E) 45°



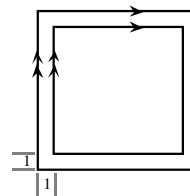
8. On a 240 kilometre trip, Corey's father drove $\frac{1}{2}$ of the distance. His mother drove $\frac{3}{8}$ of the total distance and Corey drove the remaining distance. How many kilometres did Corey drive?
- (A) 80 (B) 40 (C) 210 (D) 30 (E) 55
9. Evaluate $(-50) + (-48) + (-46) + \dots + 54 + 56$.
- (A) 156 (B) 10 (C) 56 (D) 110 (E) 162
10. The ages of three contestants in the Cayley Contest are 15 years, 9 months; 16 years, 1 month; and 15 years, 8 months. Their average (mean) age is
- (A) 15 years, 8 months (B) 15 years, 9 months (C) 15 years, 10 months
(D) 15 years, 11 months (E) 16 years

Part B: Each correct answer is worth 6.

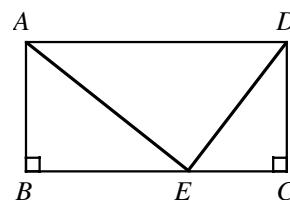
11. A store had a sale on T-shirts. For every two T-shirts purchased at the regular price, a third T-shirt was bought for \$1.00. Twelve T-shirts were bought for \$120.00. What was the regular price for one T-shirt?
- (A) \$10.00 (B) \$13.50 (C) \$14.00 (D) \$14.50 (E) \$15.00
12. Natural numbers are equally spaced around a circle in order from 1 to n . If the number 5 is directly opposite the number 14, then n is
- (A) 14 (B) 15 (C) 16 (D) 18 (E) 20
13. The average of 19 consecutive integers is 99. The largest of these integers is
- (A) 118 (B) 108 (C) 109 (D) 117 (E) 107
14. A positive integer is to be placed in each box. The product of any four adjacent integers is always 120. What is the value of x ?



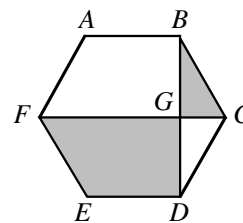
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
15. Eight squares with the same centre have parallel sides and are one unit apart. The two largest squares are shown. If the largest square has a perimeter of 96, what is the perimeter of the smallest square?



- (A) 40 (B) 68 (C) 32
(D) 64 (E) 89
16. In the diagram, $ABCD$ is a rectangle with $AD = 13$, $DE = 5$ and $EA = 12$. The area of $ABCD$ is
- (A) 39 (B) 60 (C) 52
(D) 30 (E) 25



17. In the regular hexagon $ABCDEF$, two of the diagonals, FC and BD , intersect at G . The ratio of the area of quadrilateral $FEDG$ to $\triangle BCG$ is

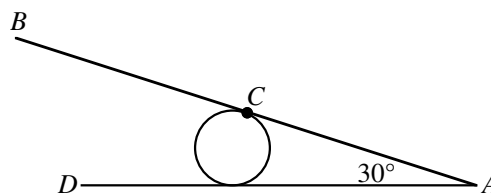


- (A) $3\sqrt{3}:1$ (B) 4:1 (C) 6:1
 (D) $2\sqrt{3}:1$ (E) 5:1

18. If a , b and c are distinct positive integers such that $abc = 16$, then the largest possible value of $a^b - b^c + c^a$ is

- (A) 253 (B) 63 (C) 249 (D) 263 (E) 259

19. A metal rod with ends A and B is welded at its middle, C , to a cylindrical drum of diameter 12. The rod touches the ground at A making a 30° angle. The drum starts to roll along AD in the direction of D . How far along AD must the drum roll for B to touch the ground?



- (A) π (B) 2π (C) 3π
 (D) 4π (E) 5π

20. Twenty pairs of integers are formed using each of the integers 1, 2, 3, ..., 40 once. The positive difference between the integers in each pair is 1 or 3. (For example, 5 can be paired with 2, 4, 6 or 8.) If the resulting differences are added together, the greatest possible sum is

- (A) 50 (B) 54 (C) 56 (D) 58 (E) 60

Part C: Each correct answer is worth 8.

21. A wooden rectangular prism has dimensions 4 by 5 by 6. This solid is painted green and then cut into 1 by 1 by 1 cubes. The ratio of the number of cubes with exactly two green faces to the number of cubes with three green faces is

- (A) 9:2 (B) 9:4 (C) 6:1 (D) 3:1 (E) 5:2

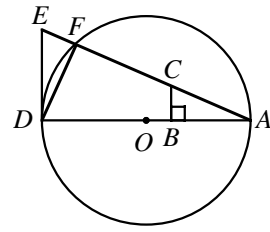
22. An ant walks inside a 18 cm by 150 cm rectangle. The ant's path follows straight lines which always make angles of 45° to the sides of the rectangle. The ant starts from a point X on one of the shorter sides. The first time the ant reaches the opposite side, it arrives at the midpoint. What is the distance, in centimetres, from X to the nearest corner of the rectangle?

- (A) 3 (B) 4 (C) 6 (D) 8 (E) 9

22. The left most digit of an integer of length 2000 digits is 3. In this integer, any two consecutive digits must be divisible by 17 or 23. The 2000th digit may be either ' a ' or ' b '. What is the value of $a + b$?

- (A) 3 (B) 7 (C) 4 (D) 10 (E) 17

24. In the diagram shown, $\angle ABC = 90^\circ$, $CB \parallel ED$,
 $AB = DF$, $AD = 24$, $AE = 25$ and O is the centre of the circle. Determine the perimeter of $CBD F$.



- (A) 39 (B) 40 (C) 42
 (D) 43 (E) 44

25. For the system of equations $x^2 + x^2y^2 + x^2y^4 = 525$ and $x + xy + xy^2 = 35$, the sum of the real y values that satisfy the equations is

- (A) 20 (B) 2 (C) $\frac{3}{2}$ (D) $\frac{55}{2}$ (E) $\frac{5}{2}$