Canadian Mathematics Competition
An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Concours canadien de mathématiques
Une activité du Centre d'éducation en mathématiques et en informatique, Université de Waterloo, Waterloo, Ontario

2001 Results

Euclid Contest

(Grade 12)

for the

The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

Awards

2001 Résultats

Concours Euclide

(12e année – Sec. V)

pour les prix

Le CENTRE d'ÉDUCATION en MATHÉMATIQUES et en INFORMATIQUE

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Foreword — Avant-Propos

The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

Le CENTRE d'ÉDUCATION en MATHÉMATIQUES et en INFORMATIQUE

K. Stephen Brown

The Centre for Education in Mathematics and Computing is pleased to have sponsored the Canadian Mathematics Competition in the year 2001. We are proud to be part of an activity that has provided mathematics enrichment to young Canadians for 38 years.

The Mathematical and Computer Sciences have enjoyed unprecedented growth, and will continue to do so into the new millennium. Continuing to study in these areas provides excellent preparation for a range of rewarding opportunities in many fields, not only in mathematics and computing. The abundance of possibilities comes not just from the technical expertise learned in these areas, but also from the problem solving, logical thinking and interpersonal skills that are inherently developed by pursuing these endeavours. The challenges are enormous, but very rewarding, and the work is fascinating, with profound implications for our society.

So congratulations to everyone who wrote this year's Euclid Contest. We hope you enjoyed the experience and we look forward to your participation in other mathematical and computing "extra-curricular" activities.

K. Stephen Brown

Director
Centre for Education in Mathematics
and Computing

Le Centre d'éducation en mathématiques et en informatique à le plaisir de commanditer le Concours canadien de mathématiques de l'année 2001. Nous sommes fiers de participer à une activité qui stimule un intérêt pour les mathématiques auprès des jeunes Canadiens depuis 38 ans.

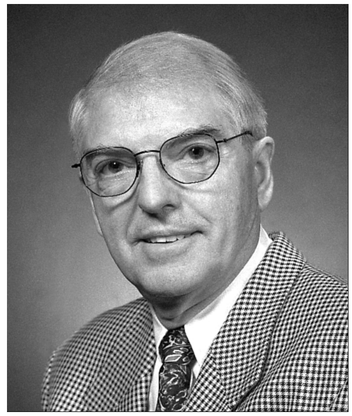
Les mathématiques et sciences informatiques ont connu un essor sans précédent et on peut s'attendre à ce qu'il en soit ainsi au cours du nouveau millénaire. Poursuivre des études dans ces secteurs du savoir fournit une excellente préparation pour un éventail d'emplois enrichissants dans un grand nombre de domaines, et pas uniquement en mathématique ou en informatique. Le grand nombre de possibilités offertes découlent non seulement des compétences techniques acquises dans ces secteurs, mais aussi de la capacité de résoudre des problèmes, d'avoir une pensée logique et d'établir des relations interpersonnelles, aptitudes qui s'acquièrent essentiellement en persistant dans ces efforts. Les défis sont énormes mais très gratifiants et le travail, qui a des répercussions profondes sur notre société, est fascinant.

Toutes nos félicitations à tous ceux qui ont participé aux concours Euclide de cette année. Nous espérons que vous avez aimé votre expérience et espérons que vous participerez à d'autres activités parascolaires liées aux mathématiques et à l'informatique.

K. Stephen Brown

Directeur
Le Centre d'Education en Mathématiques
et en Informatique

Canadian Mathematics Competition
Concours canadien de mathématiques



Ronald G. Scoins

We are pleased to post the national results of the 2001 Euclid Mathematics Contest. The CMC Executive is appreciative of the support given by the Centre for Education in Mathematics and Computing and, in particular, for providing the cash and book prizes for the Euclid contest winners.

The average score on this year's contest is 39.13, up from last year's average of 36.66. The higher average this year was almost entirely the result of many students earning marks on question 8. The last section of the paper could be classified as the "problems" section. Helping students develop their problem solving capabilities is one of the most valuable outcomes of a mathematics program. There is much to be gained by students from trying the questions again after the contest has been written. This provides an opportunity to consider alternative approaches to solving the problems. Persistence and reflection are important attributes of problem-solvers.

The Euclid Contest is based on the pre-calculus curriculum of grades 11 and 12 and requires students to write complete solutions. An emphasis on communication of mathematical ideas is very important. Many of the markers commented on how well so many of the students presented their work.

The Euclid Contest is excellent preparation for the Descartes Contest and university studies. Many of the problems are well within the reach of good grade10 and 11 students. We strongly recommend that younger students be encouraged to participate. A mathematics program that includes regular problem solving activities is the best preparation for success on the Euclid Contest.

To all students who reached their personal goals, congratulations! To students on the national prize list, to school champions, and to those who made the honour roll, well done. To all the teachers and coaches, thank you for helping make mathematics an enjoyable and rewarding enrichment activity. Your encouragement of students to strive to reach their potential is very much appreciated by the executive of the Canadian Mathematics Competition. I'm sure the students and their parents also appreciate your dedication to this goal.

Ronald G.Scoins
Executive Director

Il nous fait plaisir d'afficher les résultats nationaux du concours de mathématiques Euclide 2001. Les responsables du Concours canadien de mathématiques (CCM) sont reconnaissants envers le « Centre for Education in Mathematics and Computing » (Centre d'études en mathématiques et en informatique) pour son appui et, en particulier, pour leur avoir fourni les prix en argent et en livres remis aux gagnants du concours Euclide.

La note moyenne obtenue pour le concours de cette année est de 39,13, ce qui représente une hausse par rapport à la moyenne de l'année dernière qui était de 36,66. La plus forte moyenne enregistrée cette année est presqu'entièrement attribuable au grand nombre d'étudiants qui ont gagné des points à la question 8. La dernière section du cahier d'examen pourrait être qualifiée de section « à problèmes ». Aider les étudiants à développer leurs capacités à résoudre des problèmes est l'un des résultats les plus valorisants d'un programme de mathématiques. Les étudiants ont beaucoup à gagner à essayer de nouveau les questions après la rédaction du concours. Cela leur donne l'occasion d'envisager d'autres façons de résoudre les problèmes. La persévérance et la réflexion sont des qualités importantes chez ceux qui résolvent des problèmes.

Le Concours Euclide est basé sur le programme de cours des niveaux 11 et 12 précédant le calcul infinitésimal et exige des étudiants qu'ils fournissent par écrit des solutions complètes. Il est très important de mettre l'accent sur la communication des notions de mathématiques. Un bon nombre des correcteurs ont fait remarquer avoir été surpris de la qualité de la présentation du travail chez un si grand nombre d'étudiants.

Le Concours Euclide constitue une excellent préparation pour le Concours Descartes et les études universitaires. Un bonne partie des problèmes sont largement à la portée des bons étudiants des niveaux 10 et 11. Nous recommandons fortement que l'on encourage de plus jeunes étudiants à y participer. Un programme de mathématiques qui comporte des activités régulières de résolution de problèmes constitue la meilleur préparation pour avoir du succès au Concours Euclide.

À tous les étudiants qui ont atteint leurs objectifs personnels, félicitations! Aux étudiants dont le nom figure sur la liste nationale des prix , aux champions dans les écoles et à ceux qui se sont inscrits au tableau d'honneur, bravo! A tous les enseignants et tuteurs, merci d'aider à faire des mathématiques une activité d'enrichissement plaisante et gratifiante. Les encouragements que vous prodiguez aux étudiants pour les amener à atteindre leur plein potentiel sont grandement appréciés par les responsables du Concours canadien des mathématiques. Je suis persuadé que les étudiants et leurs parents apprécient les efforts que vous faites dans ce sens.

Ronald G. Scoins
Directeur administratif

STUDENTS / ÉLÈVES

Students are listed in alphabetical order. / Les élèves sont nommés en ordre alphabetique.

Gold Medals /	Liang Hong	University of Toronto Schools	Toronto, Ontario
Médailles d'or	Xiaoxuan Jin	Hon. Vincent Massey Secondary School	Windsor, Ontario
	Roger Mong	Don Mills Collegiate Institute	Don Mills, Ontario
	Liviu Tancau	Don Mills Collegiate Institute	Don Mills, Ontario
	Xin Zhang	Woburn Collegiate Institute	Toronto, Ontario

TEAMS / ÉQUIPES

Champion / Première :	Don Mills Collegiate Institute	Don Mills, Ontario
Second / Deuxième :	Hon. Vincent Massey Secondary School	Windsor, Ontario
Third / Troisième :	University of Toronto Schools	Toronto, Ontario
Fourth / Quatrième :	David Thompson Secondary School	Vancouver, British Columbia
	Earl Haig Secondary School	North York, Ontario
Sixth / Sixième :	Woburn Collegiate Institute	Toronto, Ontario

The Centre for Education in Mathematics and Computing Prize List / Liste des prix de la Centre d'Éducation en Mathématiques et en Informatique

Students are listed in alphabetical order. / Les élèves sont nommés en ordre alphabetique.

Cash Prizes ($500 each) / Prix en argent (500 $ chacun)

Liang Hong	University of Toronto Schools	Toronto, Ontario
Xiaoxuan Jin	Hon. Vincent Massey Secondary School	Windsor, Ontario
Roger Mong	Don Mills Collegiate Institute	Don Mills, Ontario
Liviu Tancau	Don Mills Collegiate Institute	Don Mills, Ontario
Xin Zhang	Woburn Collegiate Institute	Toronto, Ontario

Book Prizes / Prix en livres

Ron Appel	Earl Haig Secondary School	North York, Ontario
Olena Bormashenko	Don Mills Collegiate Institute	Don Mills, Ontario
Daniel Brox	Sentinel Secondary School	West Vancouver, British Columbia
Brian Choi	Markville Secondary School	Markham, Ontario
James Huang	University of Toronto Schools	Toronto, Ontario
Cornwall Lau	David Thompson Secondary School	Vancouver, British Columbia
Jeremy Nicholl	Horton High School	Wolfville, Nova Scotia
Yin Ren	Hon. Vincent Massey Secondary School	Windsor, Ontario
Alex Shyr	VSB/UBC Transition Program	Vancouver, British Columbia
Michael Tso	St. Michael's University School	Victoria, British Columbia
Shuo Xiang	Glenforest Secondary School	Mississauga, Ontario

The Canadian Mathematics Competition is grateful for the support of the Centre for Education in Mathematics and Computing in providing prizes to the top competitors in the Euclid Mathematics Contest.

Le Concours canadien de mathématiques remercie la Centre d'éducation en mathématiques et en informatique qui fournit les prix aux gagnants du Concours de mathématiques Euclide.

Please note that a student may not be a prize recipient in the Euclid Contest and in the Descartes Contest in the same year. Awards listed are at the discretion of the Executive Committee of the Canadian Mathematics Competition.

Veuillez noter qu'un étudiant ne peut recevoir un prix la même année à la fois dans le Concours Euclide et dans le Concours Descartes. Les prix indiqués le sont à la discrétion du Comité exécutif du Concours canadien de mathématiques.

1. Answers: (a) 0, 3 (b) -1, 5 (c) (4,3), (-3,-4)
Students did very well on all parts of this question. In part (c), there were several different approaches that could be used, including a graphical approach where both the circle and straight line were graphed. The average mark was 7.51.

2. Answers: (a) 3 (b) 105° (c) 59
In part (a), the key was to recognize that the form of the parabola immediately gives $b = 2$ and $b + h = 5$; most students saw this and got the right answer. Part (b) was extremely well done. There were several different approaches to part (c), including using trigonometric ratios and using similar triangles, and the majority of students answered this part correctly.
The average mark was 7.39.

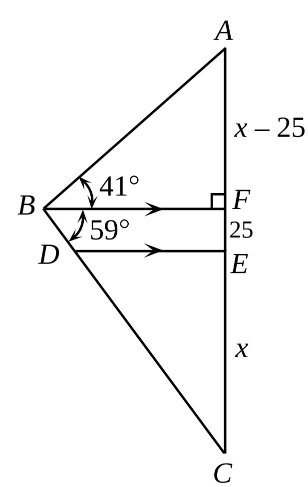
3. Answers: (a) 3 (b) Sequence 1: -2, -1, 0, 1, 2; Sequence 2: 10, 11, 12, 13, 14
In part (a), the important idea was to see that when four terms are deleted from the end of the sequence, the middle term shifts two places to the left.
Part (b) was done very well. The approach which was easiest algebraically is as follows.

Let the sequence be $a-2$, $a-1$, a, $a+1$, $a+2$, where a is an integer.

Then $$(a-2)^2+(a-1)^2+a^2=(a+1)^2+(a+2)^2$$
$$a^2-4a+4+a^2-2a+1+a^2=a^2+2a+1+a^2+4a+4$$
$$a^2-12a=0$$
$$a(a-12)=0$$

So $a=0$ or $a=12$.
Therefore, the two sequences are $-2,-1,0,1,2$ and 10, 11, 12, 13, 14. The average mark was 5.61.

4. Answers: (a) 2 (b) 79.67
Part (a) was done reasonably well. Some students did not notice that 0 is not a *positive* value for *t*, and some had some difficulty dealing with radians, but for the most part this was done well. The fastest approach was to observe that since $t>0$, $\pi t-\frac{\pi}{2}>-\frac{\pi}{2}$.
So $\sin\left(\pi t-\frac{\pi}{2}\right)$ first attains its minimum value when $\pi t-\frac{\pi}{2}=\frac{3\pi}{2}$ or $t=2$.
Part (b) was quite difficult for question 4(b). The main idea was to let the length of $AE = EC = x$, and then use trigonometric ratios to solve for x, as follows.



Then $AF = x-25$.

In, ΔBCF, $\frac{x+25}{BF}=\tan 59^\circ$.

In ΔBAF, $\frac{x-25}{BF}=\tan 41^\circ$.

Solving for BF in these two equations and equating,
$$BF=\frac{x+25}{\tan 59^\circ}=\frac{x-25}{\tan 41^\circ}$$
so $$(\tan 41^\circ)(x+25)=(\tan 59^\circ)(x-25)$$
$$25(\tan 59^\circ+\tan 41^\circ)=x(\tan 59^\circ-\tan 41^\circ)$$
$$x=\frac{25(\tan 59^\circ+\tan 41^\circ)}{\tan 59^\circ-\tan 41^\circ}$$
$$x \doteq 79.67$$
Therefore the length of AE is 79.67. The average mark was 2.17

5. Answers: (a) -1, 0, 1 (b) 3

Part (a) was done well, and reasonably easily when it is realized that x^2+5 is always positive, and so $x^2-3<0$.
Part (b) was again quite difficult, but despite this, students had a good deal of success. It does seem at first glance that there is not enough information to solve the problem. However, try letting n be the number of children. Next, we can realize that the sum of the ages of the children two years ago was $C-2n$, as each of the n children was 2 years younger. Similarly we can create two other equations. So a system of three equations in three unknowns P, C and n can be set up and solved. It is interesting to note that the actual ages of the children and the parents do not enter into the calculations.

The average mark was 4.61.

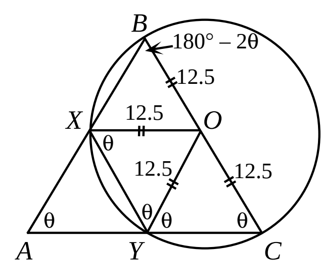
6. Answers: (a) Gold: B; Silver: A; Bronze: C (b) 15

Part (a) was one of the most attempted of any of the later questions on the paper. This question was done very well. It could be done by systematically creating a table of the coaches predictions

Medal	Gold	Silver	Bronze
Coach 1	A	B	C
Coach 2	B	C	D
Coach 3	C	A	D

and proceeding systematically from here.
There were many approaches to part (b). This helped to make this question fairly approachable. One of the most natural is the following:

Let O be the centre of the circle.
Join O to X and O to Y.
Then $OB=OC=OX=OY=12.5$ since all are radii of the circle.
Let $\angle BCA=\theta$. Then $\angle OYC=\angle BAC=\theta$ since ΔOYC and ΔBAC is isosceles.
Then $\angle ABC=180^\circ-2\theta$.
But $\angle XBC+\angle XYC=180^\circ$ since $XBCY$ is cyclic.
Thus, $\angle XYO=\theta$ and so $\angle YXO=\theta$.



Therefore, ΔABC is similar to ΔXOY which gives $\dfrac{XY}{AC}=\dfrac{OY}{BC}\Rightarrow XY=30\cdot\dfrac{12.5}{25}=15$.

The average mark was 4.02.

7. Answers: (a) 9 (b) 210

Part (a) was done reasonably well, although some students had some trouble with the idea of a base 2 logarithm. The concept behind solving for x was to raise each side to the power of 2 twice, and thus obtain a linear equation in x.
The solution to part (b) proceeds as follows.

From the given condition,

$$\frac{f(3)}{f(6)}=\frac{2^{3k}+9}{2^{6k}+9}=\frac{1}{3} \qquad (*)$$

$$3\left(2^{3k}+9\right)=2^{6k}+9$$

$$0=2^{6k}-3\left(2^{3k}\right)-18.$$

$$0=\left(2^{3k}\right)^2-3\left(2^{3k}\right)-18$$

$$0=\left(2^{3k}-6\right)\left(2^{3k}+3\right)$$

Therefore, $2^{3k}=6$ or $2^{3k}=-3$.

Since $2^a > 0$ for any a, then $2^{3k} \neq -3$.

So $2^{3k} = 6$. We could solve for k here, but this is unnecessary.

We calculate $f(9) - f(3) = \left(2^{9k} + 9\right) - \left(2^{3k} + 9\right)$

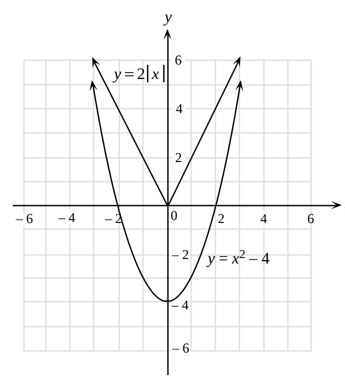
$$\begin{aligned} &= 2^{9k} - 2^{3k} \\ &= \left(2^{3k}\right)^3 - 2^{3k} \\ &= 6^3 - 6 \\ &= 210. \end{aligned}$$

Therefore $f(9) - f(3) = 210$.

Many students correctly obtained the appropriate equation (*), but then did not recognize that this was a quadratic equation in 2^{3k}, and so had some difficulty. Note k does not need to be determined in order to answer the question.

The average mark was 3.17.

8. Answers: (a) See sketch (b) $k < -5$ (c) $k = -5,\ k > -4$



The sketch in part (a) was extremely well done. It was very clear that most students have a good handle on how to graph a function involving absolute values.

Part (b) was harder, because it involved having to deal algebraically with both x and $|x|$ in the same equation. The best way to proceed is as follows.

Since each of these two graphs is symmetric about the y-axis (i.e. both are even functions), then we only need to find k so that there are no points of intersection with $x \geq 0$.

So let $x \geq 0$ and consider the intersection between $y = 2x + k$ and $y = x^2 - 4$.

Equating, we have, $2x + k = x^2 - 4$.

Rearranging, we want $x^2 - 2x - (k+4) = 0$ to have no roots x with $x \geq 0$. In fact, this quadratic equation has the sum of its roots equal to 2, so it must have at least one positive root if it has any real roots. So saying that it has no non-negative roots is the same as saying it has no real roots.

For no solutions, the discriminant is negative, i.e.

$$\begin{aligned} 20 + 4k &< 0 \\ 4k &< -20 \\ k &< -5. \end{aligned}$$

So $y = x^2 - 4$ and $y = 2|x| + k$ have no intersection points when $k < -5$.

In part (c), all that was required were the two conditions. These could be arrived at in a number of ways, including the graphical approach of “sliding” the absolute value function up and down to obtain the correct answers. The average mark was 4.38.

9. This question was hard to get a handle on. A couple of hints for how to approach this problem. Label the sides of the big triangle as a, b, c. Given the ratio of the areas of the big and little triangles, what does this say about the side lengths of the smaller triangle? Now try to set up an equation that incorporates the fact that the side lengths are all 2 units apart. One approach is to divide the big triangle into the small triangle and three trapezoids and compare areas. Using this, a Diophantine equation can be created and reduced to two variables. Try this out!

The average mark was 0.22.

10. There were many different approaches to this last question, ranging from a couple of lines long to a couple of pages long! Again, a couple of hints to obtain a quick solution. Join D to B. What can be said about the way the point of intersection, R, of BD and PQ? Try using the sine law in triangle DPR and see what can concluded about the *maximum* of angle PDR. How does this help us with the *minimum* of angle ADP? Good luck working on this! The average mark was 0.03.

Note: Full solutions to 9 and 10 will be posted here on June 25, 2001.