Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

# Pascal Contest (Grade 9)

## Wednesday, February 20, 2002

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**Time:** 1 hour



**Calculators are permitted**, providing they are non-programmable and without graphic displays.

**Instructions**

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

| Scoring: | There is *no penalty* for an incorrect answer.<br>Each unanswered question is worth 2, to a maximum of 10 unanswered questions. |
|---|---|

**Part A: Each correct answer is worth 5.**

1. $\dfrac{15+9-6}{3\times 2}$ equals

   (**A**) 11 (**B**) 4 (**C**) 3 (**D**) 23 (**E**) 12

2. 50% of 2002 is equal to

   (**A**) 4004 (**B**) 3003 (**C**) 2001 (**D**) 1952 (**E**) 1001

3. If $x+2=10$ and $y-1=6$, then the numerical value of $x+y$ is

   (**A**) 13 (**B**) 15 (**C**) 16 (**D**) 17 (**E**) 19

4. The value of $\left(3^2-3\right)^2$ is

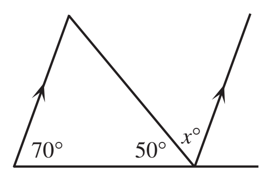
   (**A**) 36 (**B**) 72 (**C**) 9 (**D**) 3 (**E**) 0

5. Sofia entered an elevator. The elevator went up seven floors, then down six floors, and finally up five floors. If Sofia got out on the twentieth floor, then she entered the elevator on floor number

   (**A**) 14 (**B**) 2 (**C**) 16 (**D**) 38 (**E**) 26

6. In the diagram, the value of $x$ is

   (**A**) 20 (**B**) 60 (**C**) 70
   (**D**) 40 (**E**) 50




7. If $n$ is $\frac{5}{6}$ of 240, then $\frac{2}{5}$ of $n$ is

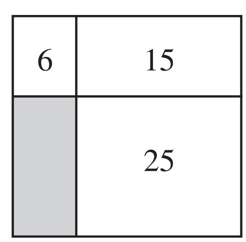
   (**A**) 288 (**B**) 80 (**C**) 96 (**D**) 200 (**E**) 500

8. The value of $1-\left(5^{-2}\right)$ is

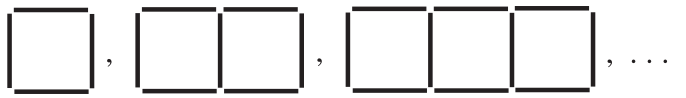
   (**A**) $\frac{24}{25}$ (**B**) $-24$ (**C**) $\frac{26}{25}$ (**D**) 26 (**E**) $\frac{9}{10}$

9. A rectangle is divided into four smaller rectangles. The areas of three of these rectangles are 6, 15 and 25, as shown. The area of the shaded rectangle is

   (**A**) 7 (**B**) 15 (**C**) 12
   (**D**) 16 (**E**) 10



10. Toothpicks are used to form squares in the pattern shown:



Four toothpicks are used to form one square, seven to form two squares, and so on. If this pattern continues, how many toothpicks will be used to form 10 squares in a row?

(**A**) 39 (**B**) 40 (**C**) 31 (**D**) 35 (**E**) 28

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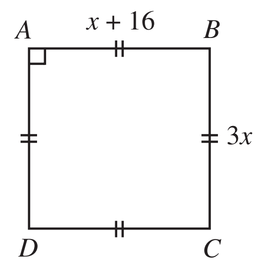
**Part B: Each correct answer is worth 6.**

11. $ABCD$ is a square with $AB = x + 16$ and $BC = 3x$, as shown. The perimeter of $ABCD$ is

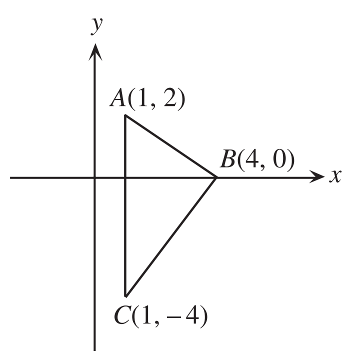



(**A**) 16 (**B**) 32 (**C**) 96
(**D**) 48 (**E**) 24

12. In a sequence of numbers, each number, except the first, equals twice the previous number. If the sum of the second and third numbers in the list is 24, then the *sixth* number is

(**A**) 112 (**B**) 192 (**C**) 64 (**D**) 40 (**E**) 128

13. Triangle $ABC$ has vertices $A(1,2)$, $B(4,0)$ and $C(1,-4)$. The area of $\Delta ABC$ is

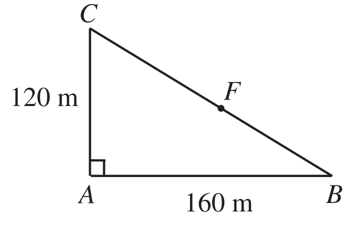



(**A**) 18 (**B**) 12 (**C**) 8
(**D**) 10 (**E**) 9

14. A class of 30 students wrote a history test. Of these students, 25 achieved an average of 75%. The other 5 students achieved an average of 40%. The class average on the history test was closest to

(**A**) 46 (**B**) 69 (**C**) 63 (**D**) 58 (**E**) 71

15. In the diagram, $ABC$ represents a triangular jogging path. Jack jogs along the path from $A$ to $B$ to $F$. Jill jogs from $A$ to $C$ to $F$. Each jogs the same distance. The distance from $F$ to $B$, in metres, is




(**A**) 40 (**B**) 120 (**C**) 100
(**D**) 80 (**E**) 200

16. When the product $\left(5^3\right)\left(7^{52}\right)$ is expanded, the units digit (that is, the last digit) is

(**A**) 5 (**B**) 3 (**C**) 9 (**D**) 7 (**E**) 0

17. The number 1000 can be written as the product of two positive integers, neither of which contains zeros. The sum of these two integers is

(**A**) 65 (**B**) 110 (**C**) 133 (**D**) 205 (**E**) 1001

18. Together Akira and Jamie weigh 101 kg. Together Akira and Rabia weigh 91 kg. Together Rabia and Jamie weigh 88 kg. How many kilograms does Akira weigh?

(**A**) 48 (**B**) 46 (**C**) 50 (**D**) 52 (**E**) 38

19. The natural numbers from 1 to 2100 are entered sequentially in 7 columns, with the first 3 rows as shown. The number 2002 occurs in column $m$ and row $n$. The value of $m+n$ is

| | Column 1 | Column 2 | Column 3 | Column 4 | Column 5 | Column 6 | Column 7 |
|---|---|---|---|---|---|---|---|
| Row 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Row 2 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| Row 3 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

(**A**) 290 (**B**) 291 (**C**) 292 (**D**) 293 (**E**) 294

20. For how many integer values of $x$ is $\sqrt{25-x^2}$ equal to an integer?

(**A**) 7 (**B**) 6 (**C**) 5 (**D**) 3 (**E**) 2

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**Part C: Each correct answer is worth 8.**

21. A rectangular block, with dimensions 4 cm, 5 cm and 6 cm, is made up of cubes each with side length 1 cm. If 1 $cm^3$ cubes are removed from this larger rectangular block, what is the minimum number of these cubes that must be removed so that the resulting solid is itself a cube?

(**A**) 40 (**B**) 93 (**C**) 46 (**D**) 64 (**E**) 56

22. In a school, 500 students voted on each of two issues. Of these students, 375 voted in favour of the first issue, 275 voted in favour of the second, and 40 students voted against both issues. How many students voted in favour of both issues?

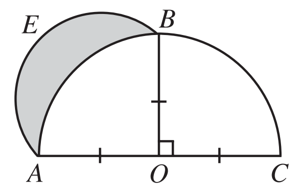
(**A**) 110 (**B**) 150 (**C**) 190 (**D**) 95 (**E**) 230

23. The number of ordered pairs $(a,b)$ of *integers* which satisfy the equation $a^b = 64$ is

(**A**) 3 (**B**) 5 (**C**) 8 (**D**) 6 (**E**) 7

continued ...

24. In the diagram, $ABC$ is a semi-circle with diameter $AC$, centre $O$ and radius 1. Also, $OB$ is perpendicular to $AC$. Using $AB$ as a diameter, a second semi-circle $AEB$ is drawn. The region inside this second semi-circle that lies outside the original semi-circle is shaded, as shown. The area of this shaded region is




(**A**) $\frac{\pi}{4}$ (**B**) $\frac{1}{2}$ (**C**) $\frac{3\pi}{4}+\frac{1}{2}$

(**D**) $\frac{3}{4}$ (**E**) $\frac{\pi}{2}-\frac{1}{2}$

25. A student has two open-topped cylindrical containers. (The walls of the two containers are thin enough so that their width can be ignored.) The larger container has a height of 20 cm, a radius of 6 cm and contains water to a depth of 17 cm. The smaller container has a height of 18 cm, a radius of 5 cm and is empty. The student slowly lowers the smaller container into the larger container, as shown in the cross-section of the cylinders in Figure 1. As the smaller container is lowered, the water first overflows out of the larger container (Figure 2) and then eventually pours into the smaller container. When the smaller container is resting on the bottom of the larger container, the depth of the water in the smaller container will be closest to

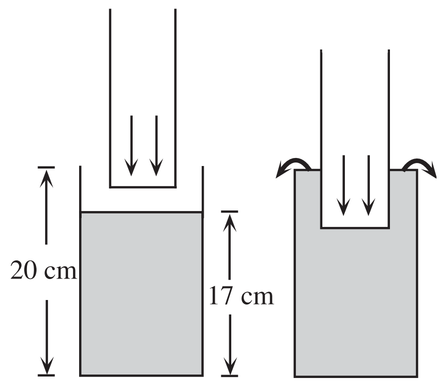


Figure 1 Figure 2

(**A**) 2.82 cm (**B**) 2.84 cm (**C**) 2.86 cm

(**D**) 2.88 cm (**E**) 2.90 cm