Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

2003 Solutions

Gauss Contest

(Grades 7 and 8)

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Part A

1. Multiplying gives $3.26 \times 1.5 = 4.89$. ANSWER: (B)

2. Calculating in the parentheses first,

$$(9-2)-(4-1) = 7-3 = 4.$$

ANSWER: (C)

3. Adding gives,

$$30 + 80\,000 + 700 + 60 = 30 + 80\,760 = 80\,790$$

ANSWER: (D)

4. $\dfrac{1+2+3}{4+5+6} = \dfrac{6}{15} = \dfrac{2}{5}$ ANSWER: (C)

5. In the survey, a total of 90 people were surveyed.
According to the graph, 25 people chose a cat, 10 people chose a fish, 15 people chose a bird, and 5 people chose "other", accounting for $25+10+15+5=55$ people.
This leaves $90-55=35$ people who have chosen a dog. ANSWER: (E)

6. If Travis uses 4 mL of gel every day and a tube of gel contains 128 mL of gel, then it will take him $\dfrac{128}{4} = 32$ days to empty the tube. ANSWER: (A)

7. *Solution 1*
On the left hand side of the equation, if the 3's are cancelled, we would have

$$\frac{3\times 6\times 9}{3} = 6\times 9 = \frac{\square}{2}$$

Therefore, the expression in the box should be $2\times 6\times 9$, since we can write $6\times 9 = \dfrac{2\times 6\times 9}{2}$.

Solution 2
Evaluating the left side of the expression, we obtain $\dfrac{3\times 6\times 9}{3} = 54$.
Therefore, we must place an expression equal to $54\times 2 = 108$ in the box to make the equation true.
Evaluating the five choices, we obtain
(A) 48 (B) 72 (C) 108 (D) 64 (E) 432
Therefore, the answer must be $2\times 6\times 9$. ANSWER: (C)

8. If we turn the words

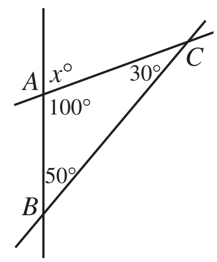
ƎJAƧ ЯOꟻ ᗡƆ ꓘИUꟼ

around and look at them as if looking at them through the opposite side of a window, the only letters that appear the same will be U, O, and A. The other letters will all appear differently from the other side of the window. ANSWER: (A)

9. Spencer starts 1000 m from home and walks to a point 800 m from home, a distance of 200 m. He then walks to a point 1000 m from home, for a distance of another 200 m. Finally, he walks home, a distance of 1000 m. So Spencer has walked a total of $200 + 200 + 1000 = 1400$ m.

ANSWER: (E)

10. Since the sum of the angles in a triangle is 180°, then $\angle BAC = 180^\circ - 30^\circ - 50^\circ = 100^\circ$.
Since a straight line makes an angle of 180°, then $x^\circ + 100^\circ = 180^\circ$ or $x = 80$.



ANSWER: (A)

Part B

11. Since there are 12 squares initially, then the number of squares to be removed is
$\frac{1}{2} \times \frac{2}{3} \times 12 = \frac{1}{3} \times 12 = 4$
Therefore, there will be 8 squares remaining.

ANSWER: (D)

12. *Solution 1*
Since the perimeter of the field is 3 times the length and the perimeter is 240 m, then the length of the field is 80 m.
Since the perimeter of a rectangle is two times the length plus two times the width, then the length accounts for 160 m of the perimeter, leaving 80 m for two times the width.
Therefore, the width of the field is 40 m.

Solution 2
Let the perimeter of the field be P, the length be l, and the width w.
We are given that $P = 3l$ and $P = 240$, so $l = \frac{1}{3}(240) = 80$.
Since $P = 240$ and $P = 2l + 2w$, we have $240 = 2(80) + 2w$ or $w = 40$.

ANSWER: (B)

13. Since Chris runs $\frac{1}{2}$ as fast as his usual running speed, he runs at 5 km/h, and so will take 6 hours to complete the 30 km run.
Since Pat runs at $1\frac{1}{2}$ her usual running speed, she runs at 15 km/h, and so will take 2 hours to complete the 30 km run.
Thus, it takes Chris 4 hours longer to complete the run than it takes Pat.

ANSWER: (D)

14. *Solution 1*

Since there are twice as many red disks as green disks and twice as many green disks as blue disks, then there are four times as many red disks as blue disks.
So the total number of disks is seven times the number of blue disks (since the numbers of red and green disks are four and two times the number of blue disks).
Since there are 14 disks in total, there are 2 blue disks, and so there are 4 green disks.

Solution 2

Let the number of green disks be g.
Then the number of red disks is $2g$, and the number of blue disks is $\frac{1}{2}g$.
From the information given,

$$\begin{aligned} 2g + g + \tfrac{1}{2}g &= 14 \\ \tfrac{4}{2}g + \tfrac{2}{2}g + \tfrac{1}{2}g &= 14 \\ \tfrac{7}{2}g &= 14 \\ g &= \tfrac{2}{7} \times 14 \\ g &= 4 \end{aligned}$$

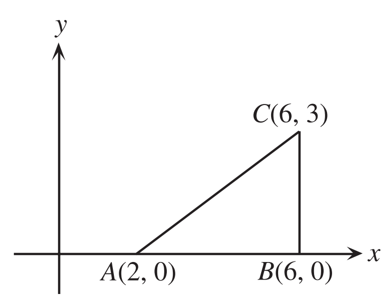
Therefore, the number of green disks is 4. ANSWER: (B)

15. In the bottle, there are a total of 180 tablets.
Among the 60 stars, there are an equal number of each of the three flavours – strawberry, grape and orange. This tells us that there are 20 grape stars.
If each tablet is equally likely to be chosen from the bottle, the probability of choosing a grape star is

$$\frac{\text{Number of grape stars}}{\text{Total number of tablets}} = \frac{20}{180} = \frac{1}{9}$$

ANSWER: (A)

16. First, we sketch the triangle.
Since point C is directly above point B (that is, angle ABC is a right angle), then we can look at triangle ABC as having base AB (of length 4) and height BC (of length 3).
Thus, the area of the triangle is $A = \frac{1}{2}bh = \frac{1}{2}(4)(3) = 6$ square units.



ANSWER: (C)

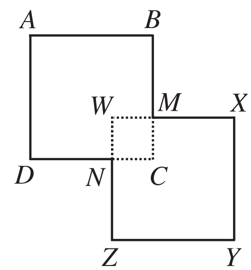
17. If Genna's total bill was \$74.16 and a \$45 fee was charged, this means that her cost based on the distance she drove was $\$74.16 - \$45.00 = \$29.16$. Thus the number of kilometres driven is $\frac{29.16}{0.12} = 243$.
ANSWER: (C)

18. *Solution 1*

We can calculate the perimeter of the shaded figure by adding the perimeters of the two large squares and subtracting the perimeter of the small square. This is because all of the edges of the two larger squares are included in the perimeter of the shaded figure except for the sides of the smaller square.
Since the side length of the larger squares is 5 cm, the perimeter of each of the two larger squares is 20 cm.
Since the area of the smaller square is 4 cm^2, then its side length is 2 cm, and so its perimeter is 8 cm.
Therefore, the perimeter of the shaded figure is $20+20-8=32$ cm.

Solution 2

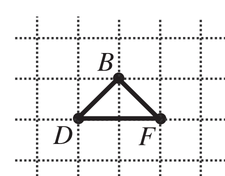
We label the vertices in the diagram and calculate directly.
We know that the two large squares each have a side length of 5 cm, so $AB=AD=YX=YZ=5$.
The smaller square has an area of 4 cm^2, and so has a side length of 2 cm. Therefore, $WM=MC=CN=NW=2$, and so each of BM, DN, MX, and NZ have a length of 3 cm (the difference between the side length of the two squares).
Thus, the total perimeter of the shaded figure is $4\times5+4\times3=32$ cm.



ANSWER: (B)

19. Abraham's exam had a total of 80 questions. Since he received a mark of 80%, he got $\frac{80}{100}\times80=\frac{8}{10}\times80=64$ questions correct.
We also know that Abraham answered 70% of the 30 algebra questions correctly, or a total of 21 questions.
This tells us that he answered 43 of the geometry questions correctly. ANSWER: (A)

20. We first make a list of all of the possible triangles with an edge lying on *DEF* (that is, with an edge that is one of *DE*, *DF* or *EF*):
DAE, *DBE*, *DCE*, *DAF*, *DBF*, *DCF*, *EAF*, *EBF*, *ECF*
Of these, triangles *DAE*, *DBE*, *DAF*, *DCF*, *EBF*, *ECF* are obviously right-angled, while triangles *DCE* and *EAF* are not right-angled because they each contain an angle of 135°. What about triangle *DBF*?
In fact, triangle *DBF* is right-angled since $\angle DBE=\angle EBF=45^\circ$, and so $\angle DBF=90^\circ$.
Therefore, there are 2 triangles among this group which are not right-angled.



If we now look at the possible triangles with an edge lying on *ABC*, we will again find 2 non right-angled triangles among these triangles. (We can see this by reflecting all of the earlier triangles to move their base from *DEF* to *ABC*.)
We can also see that it is impossible to make a triangle without an edge lying on *ABC* or *DEF*, since the only other possible edges are *AD*, *AE*, *AF*, *BD*, *BE*, *BF*, *CD*, *CE*, and *CF*. No three of these nine edges can be joined to form a triangle since each edge has one end at *A*, *B* or *C* and the other end at *D*, *E* or *F*, and there are no edges joining two points among either set of three points.
Therefore, there are only 4 triangles that can be formed that are not right-angled.

ANSWER: (E)

Part C

21. Since there are ten people scheduled for an operation and each operation begins 15 minutes after the previous one, the tenth operation will begin nine 15 minute intervals after the first operation began at 8:00 a.m.
Nine 15 minute intervals is 135 minutes, or 2 hours 15 minutes.
Thus, the tenth 45 minute operation begins at 10:15 a.m. and so ends at 11:00 a.m.

ANSWER: (D)

22. *Solution 1*

Since Luke has a 95% winning percentage, then he hasn't won 5%, or $\frac{1}{20}$, of his games to date. Since he has played only 20 games, there has only been 1 game that he has not won.

For Luke to have exactly a 96% winning percentage, he must not have won 4%, or $\frac{1}{25}$, of his games. Since he wins every game between these two positions, when he has the 96% winning percentage, he has still not won only 1 game. Therefore, he must have played 25 games in total, or 5 more than initially.

Solution 2

Since Luke has played 20 games and has a 95% winning percentage, then he has won $\frac{95}{100}\times 20 = \frac{95}{5} = 19$ games.

Let the number of games in a row he wins before reaching the 96% winning percentage be x. Then

$$\text{Winning } \% = \frac{\text{Games Won}}{\text{Games Played}} = \frac{19+x}{20+x} = \frac{96}{100}$$

Cross-multiplying,

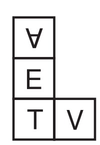
$$\begin{aligned} 100(19+x) &= 96(20+x) \\ 1900+100x &= 1920+96x \\ 4x &= 20 \\ x &= 5 \end{aligned}$$

Therefore, we wins 5 more games in a row.

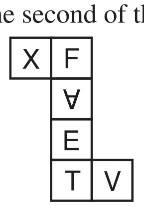
ANSWER: (D)

23. We will determine which letter belongs on the shaded face by "unfolding" the cube.
Using the first of the three positions, we obtain

Using the third of the three positions, we can add the "A" above the "E" as



Using the second of the three positions, the F can be placed above the A and the X to the left of the F to get



If we refold this cube, with A at the front, F on top, and E on the bottom, the right-hand face will be the V (upside down), and so the shaded face is the V. ANSWER: (E)

24. To get a better understanding of the pattern, let us write each of the numbers in the way that it is obtained:

$$\begin{array}{ccccccccc} & & & 1 & & 2 & & & \\ & & 1 & & 3 & & 2 & & \\ & 1 & & 4 & & 5 & & 2 & \\ 1 & & 5 & & 9 & & 7 & & 2 \\ \cdot^{\cdot^{\cdot}} & & & & \vdots & & & & \ddots \end{array} \qquad \begin{array}{ccccccccc} & & & 1 & & 2 & & & \\ & & 1 & & 1+2 & & 2 & & \\ & 1 & & 1+3 & & 3+2 & & 2 & \\ 1 & & 1+4 & & 4+5 & & 5+2 & & 2 \\ \cdot^{\cdot^{\cdot}} & & & & \vdots & & & & \ddots \end{array}$$

We can see from these two patterns side by side that each number in a row is accounted for *twice* in the row below. (The 1 or 2 on the end of a row appears again at the end of the row and as part of the sum in one number in the next row. A number in the middle of a row appears as part of the sum in two numbers in the next row.)
Therefore, the sum of the numbers in a row should be two times the sum of the numbers in the previous row.
We can check this:

Sum of the numbers in the 1st row	3
Sum of the numbers in the 2nd row	6
Sum of the numbers in the 3rd row	12
Sum of the numbers in the 4th row	24

Thus, the sum of the numbers in the thirteenth row should be the sum of the elements in the first row multiplied by 2 twelve times, or $3\times 2^{12} = 3\times 4096 = 12\ 288$. ANSWER: (D)

25. We will present a complete consideration of all of the cases. The answer can be obtained more easily in a trial and error fashion.

First, we rewrite the equation putting letters in each of the boxes

$$\begin{array}{ccc} & A & B \\ & \times & C \\ \hline D & E & F \end{array}$$

We want to replace *A*, *B*, *C*, *D*, *E*, and *F* by the digits 1 through 6.

Could *C* be 1?

If *C* was 1, then *B* and *F* would have to be same digit, which is impossible since all of the digits are different. Therefore, *C* cannot be 1.

Could *C* be 5?

If *C* was 5, then if *B* were odd, *F* would also be 5, which would be impossible. If *C* was 5 and *B* even, then *F* would have to be 0, which is also impossible. Therefore, *C* cannot be 5.

Could C be 6?

If C was 6, let us list the possibilities for B and the resulting value of F:

B	F	
1	6	Impossible – two 6's
2	2	Impossible – two 2's
3	8	Impossible – no 8
4	4	Impossible – two 4's
5	0	Impossible – no 0

Therefore, C cannot be 6.

Could C be 4?

If C was 4, using a similar chart, we can see that B must be 3 and F must be 2. We will consider this possibility later.

Could C be 3?

If C was 4, using a similar chart, we can see that B must be 2 or 4 and F must be thus 6 or 2.

Could C be 2?

If C was 2, using a similar chart, we can see that B must be 3 and F must be 6.

Let us consider the case of $C = 2$. In this case, we have

$$\begin{array}{r} A\ 3 \\ \times\ 2 \\ \hline D\ E\ 6 \end{array}$$

Since the product has three digits, and the possibilities remaining for A are 1, 4 or 5, then A must be 5. However, this gives a product of $53 \times 2 = 106$, which is impossible.

Similarly, trying the case of $C = 4$, we are left with the possibilities for A being 1, 5, or 6, none of which work.

Therefore, C must be 3, since this is the only possibility left. We should probably check that we can actually get the multiplication to work, though!

Trying the possibilities as above, we can eventually see that $54 \times 3 = 162$ works, and so $C = 3$.

ANSWER: (B)

Part A

1. Performing the calculation,

$1.000+0.101+0.011+0.001=1.113$ Answer: (B)

2. We group the terms and start with addition, and then do subtraction:

$$\begin{aligned}&1+2+3-4+5+6+7-8+9+10+11-12\\&=6-4+18-8+30-12\\&=2+10+18\\&=30\end{aligned}$$

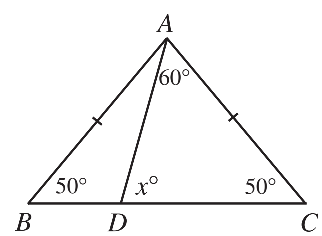
Answer: (A)

3. The amount that each charity received was the total amount raised divided by the number of charities, or $\$3109 \div 25 = \$124.36$. Answer: (E)

4. The square of the square root of 17 is $\left(\sqrt{17}\right)^2$, which equals 17. The operations of squaring and taking a square root are inverses of each other. Answer: (C)

5. Since triangle ABC is isosceles, $\angle ACB = \angle ABC = 50^\circ$.
Since the sum of the angles in triangle ACD is 180°, then

$$\begin{aligned}x^\circ + 60^\circ + 50^\circ &= 180^\circ\\ x &= 70\end{aligned}$$



Answer: (A)

6. *Solution 1*
If we "undo" the operations, we would get back to the original number by first subtracting 13 and then dividing by 2, to obtain $\frac{1}{2}(89-13)=38$.

Solution 2
Let the number be x. Then $2x+13=89$ or $2x=76$ or $x=38$. Answer: (D)

7. The range of temperature is the difference between the high temperature and the low temperature. We can complete the chart to determine the largest range.

Day	Temperature Range (°C)
Monday	$5-(-3)=8$
Tuesday	$0-(-10)=10$
Wednesday	$-2-(-11)=9$
Thursday	$-8-(-13)=5$
Friday	$-7-(-9)=2$

We see that the temperature range was greatest on Tuesday. Answer: (B)

8. We write each of the five numbers as a decimal in order to be able to arrange them in order from smallest to largest:

$$\sqrt{5} = 2.236...$$
$$2.1 = 2.1$$
$$\tfrac{7}{3} = 2.333...$$
$$2.0\overline{5} = 2.055...$$
$$2\tfrac{1}{5} = 2.2$$

So in order from smallest to largest, we have $2.0\overline{5}$, 2.1, $2\frac{1}{5}$, $\sqrt{5}$, $\frac{7}{3}$. The number in the middle is $2\frac{1}{5}$.

ANSWER: (E)

9. Since one-third of the 30 students in the class are girls, then 10 of the students are girls. This means that 20 of the students are boys. Three-quarters of the 20 boys play basketball, which means that 15 of the boys play basketball. ANSWER: (E)

10. We rewrite the addition in columns, and write each number to three decimals:

$$\begin{array}{r} 15.200 \\ 1.520 \\ 0.15\square \\ +\ \square.128 \\ \hline 20.000 \end{array}$$

Since the sum of the digits in the last column ends in a 0, then the box in the thousandths column must represent a 2.
We can insert the 2 to get

$$\begin{array}{r} 15.200 \\ 1.520 \\ 0.152 \\ +\ \square.128 \\ \hline 20.000 \end{array}$$

If we then perform the addition of the last three columns, we will get a carry of 1 into the units column. Since the sum of the units column plus the carry ends in a 0, then the box in the units column must represent a 3. Therefore, the sum of the digits inserted into the two boxes is 5. (We also check that $15.2 + 1.52 + 0.152 + 3.128 = 20$.) ANSWER: (A)

Part B

11. Reading the data from the graph the numbers of female students in the five classes in order are 10, 14, 7, 9, and 13. The average number of female students is $\frac{10+14+7+9+13}{5} = \frac{53}{5} = 10.6$.

ANSWER: (E)

12. The area of the original photo is $20 \times 25 = 500 \text{ cm}^2$ and the area of the enlarged photo is $25 \times 30 = 750 \text{ cm}^2$. The percentage increase in area is

$$\frac{\text{Final Area} - \text{Initial Area}}{\text{Initial Area}} \times 100\% = \frac{750-500}{500} \times 100\% = \frac{250}{500} \times 100\% = 50\%$$

ANSWER: (B)

13. Since the angles are in the ratio 2 : 3 : 4, then we can represent the angles as $2x$, $3x$, and $4x$ (in degrees) for some number x. Since these three angles are the angles of a triangle then

$$\begin{aligned} 2x + 3x + 4x &= 180^\circ \\ 9x &= 180^\circ \\ x &= 20^\circ \end{aligned}$$

Thus the largest angle is $4x$, or 80°.

ANSWER: (C)

14. *Solution 1*

Since George recorded his highest mark higher than it was, then if he writes his seven marks in order from lowest to highest, the order will not be affected.
So the minimum test mark is not affected, nor is the median test mark (which is the middle of the seven different marks).
Is either of the range or mean affected?
The range is difference between the highest mark and the lowest mark, so by recording the highest mark higher, the range has become larger.
The mean is the sum of the seven marks divided by 7, so since the highest mark is higher, the sum of the 7 marks will be higher, and so the mean will be higher.
So only the mean and range are affected.

Solution 2

Suppose that George's marks were 80, 81, 82, 83, 84, 85, and 86, but that he wistfully recorded the 86 as 100.
Originally, with marks 80, 81, 82, 83, 84, 85, 86, the statistics are

Mean	$\frac{80+81+82+83+84+85+86}{7} = 83$
Median	83
Minimum test score	80
Range	$86 - 80 = 6$

With marks 80, 81, 82, 83, 84, 85, 100, the statistics are

Mean	$\dfrac{80+81+82+83+84+85+100}{7}=85$
Median	83
Minimum test score	80
Range	$100-80=20$

So the mean and range are the only statistics altered. ANSWER: (C)

15. The volume of the entire pit is

$$(10\text{ m})\times(50\text{ cm})\times(2\text{ m})=(10\text{ m})\times(0.5\text{ m})\times(2\text{ m})=10\text{ m}^3$$

Since the pit starts with 5 m^3 in it, an additional 5 m^3 of sand is required to fill it.

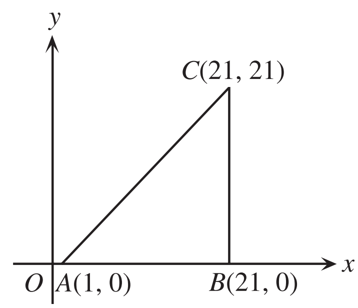
ANSWER: (B)

16. We evaluate this "continued fraction" step by step

$$\frac{1}{1+\frac{1}{1+\frac{1}{2}}}=\frac{1}{1+\frac{1}{\left(\frac{3}{2}\right)}}=\frac{1}{1+\frac{2}{3}}=\frac{1}{\left(\frac{5}{3}\right)}=\frac{3}{5}$$

ANSWER: (A)

17. The perimeter of the triangle is the sum of the lengths of the sides. We make a sketch of the triangle to help us with our calculations. Since side AB is along the x-axis and side BC is parallel to the y-axis, then the triangle is right-angled, and we can use Pythagoras' Theorem to calculate the length of AC.



The length of AB is 20 and the length of BC is 21.
Calculating AC,

$$AC^2=AB^2+BC^2$$
$$AC^2=20^2+21^2$$
$$AC^2=400+441$$
$$AC^2=841$$
$$AC=29$$

Therefore, the perimeter is $20+21+29=70$. ANSWER: (A)

18. *Solution 1*

If $-3x^2<-14$, then $3x^2>14$ or $x^2>\frac{14}{3}=4\frac{2}{3}$.

Since we are only looking at x being a whole number, then x^2 is also a whole number. Since x^2 is a whole number and x^2 is greater than $4\frac{2}{3}$, then x^2 must be at least 5.

Of the numbers in the set, the ones which satisfy this condition are $-5,-4,-3$, and 3.

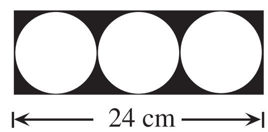
Solution 2

For each number in the set, we substitute the number for x and calculate $-3x^2$:

x	$-3x^2$
-5	-75
-4	-48
-3	-27
-2	-12
-1	-3
0	-3
0	-12

As before, the four numbers $-5, -4, -3$, and 3 satisfy the inequality. ANSWER: (D)

19. Since three circles touch each other and touch the vertical and horizontal sides of the rectangle, then the width of the rectangle is three times the diameter of the circle, and the height of the rectangle is equal to the diameter of the circle.



Since the width of the rectangle is 24 cm, then the diameter of each circle is 8 cm.
Since the diameter of each circle is 8 cm, then the height of the rectangle is 8 cm, and the radius of each circle is 4 cm.
Therefore, the area of the shaded region is

Area of shaded region
= Area of rectangle − Area of three circles
$= (24 \times 8) - 3\left[\pi(4)^2\right]$
$= 192 - 48\pi$
$\approx 192 - 150.80$ (using $\pi \approx 3.14$)
$= 41.20$

Thus, the area of the shaded region is closest to 41 cm^2. ANSWER: (A)

20. This question is made more difficult by the fact that there are 2 letters that are the same. To overcome this problem, let us change the second S to a T, so that the letters on the tiles are now G, A, U, S, and T, and we want to calculate the probability that Amy chooses the S and the T when she chooses two tiles.
Let us say that Amy chooses the tiles one at a time. How many choices does she have for the first tile she chooses? Since there are 5 tiles, she has 5 choices. How many choices does Amy have for the second tile? There are 4 tiles left, so she has 4 choices. Now for *each* of the 5 ways of choosing the first tile, she has 4 ways of choosing the second tile, for a total of 20 possibilities. (Try writing out the 20 possibilities and convince yourself that it is 5×4 and not $5 + 4$.)
In these 20 pairs, there are two pairs that contain an *S* and *T*. If the *T* was changed back to an *S*, there would be two pairs out of 20 containing 2 S's so the probability would be $\frac{2}{20} = \frac{1}{10}$ of selecting the 2 S's. ANSWER: (D)

Part C

21. Let's look at a couple of examples of four consecutive whole numbers that add to a multiple of 5, and see what possibilities we can eliminate.
First, we can look at 1, 2, 3, 4 (whose sum is 10).
Using this example, we can eliminate choice (A) (since the sum ends in a 0) and choice (B) (since the largest number ends in a 4).
With a bit more work, we can see that 6, 7, 8, 9 (whose sum is 30) is another example.
Using this second example, we can eliminate choice (C) (since the smallest number is even) and choice (E) (since none of the numbers ends in a 3).
Therefore, the only remaining choice is (D).

Why is (D) always true? This is a bit more tricky to figure out, and requires some algebra.
Let the smallest of the numbers be n. Then the other three numbers are $n+1$, $n+2$, and $n+3$, and their sum is $n+n+1+n+2+n+3=4n+6$.
We know that their sum is a multiple of 5. Since the sum is also $4n+6$, this sum is even and so must end in a 0 if it is also to be a multiple of 5.
Since $4n+6$ ends in a 0, then $4n$ ends in a 4. What can the units digit of n be? The only possibilities for the units digit of n are 1 and 6, and so the four numbers either end with 1, 2, 3, and 4, or 6, 7, 8, and 9, and none of these four numbers is a multiple of 5. ANSWER: (D)

22. *Solution 1*
When Carmina trades her nickels for dimes and her dimes for nickels, she gains \$1.80. Since trading a dime for a nickel results in a loss of 5 cents, and trading a nickel for a dime results in a gain of 5 cents, then by doing her trade she gains 5 cents in $\frac{180}{5}=36$ more cases than she loses 5 cents. Thus, she must have 36 more nickels than dimes.
These extra 36 nickels account for \$1.80. So her initial coins are worth \$1.80 and she has an equal number of nickels and dimes. A nickel and dime together are worth 15 cents, so she must have $\frac{180}{15}=12$ sets of a nickel and dime, or 12 nickels and 12 dimes.
So in total, Carmina has 48 nickels and 12 dimes, or 60 coins.

Solution 2
Suppose that Carmina has n nickels and d dimes.
Then looking at the total number of cents Carmina has, $5n+10d=360$.
If we reverse the nickels and dimes and again look at the total number of cents that Carmina has, we see that $10n+5d=540$.
If we add these two equations together, we get

$$\begin{aligned}5n+10d+10n+5d&=360+540\\15n+15d&=900\\n+d&=60\end{aligned}$$

So the total number of coins is 60. (Notice that we didn't in fact have to calculate the number of nickels or the number of dimes!)

Solution 3

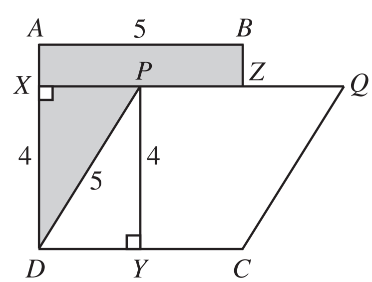
Suppose that Carmina has n nickels and d dimes.
Then looking at the total number of cents Carmina has, $5n+10d=360$ or $n+2d=72$ or $n=72-2d$.
If we reverse the number of nickels and the number of dimes and look at the number of cents,

$$\begin{aligned} 5d+10n &= 540 \\ 5d+10(72-2d) &= 540 \\ 5d+720-20d &= 540 \\ 180 &= 15d \\ d &= 12 \end{aligned}$$

Since $d=12$, then $n=72-2d=48$.
Therefore, the total number of nickels and dimes that Carmina has is 60. ANSWER: (D)

23. Suppose that Gabriella's twelve plants had 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 tomatoes. How many tomatoes would she have in total? Adding these numbers up, she would have 78 tomatoes.
But we know that she has 186 tomatoes, so there are 108 tomatoes unaccounted for.
Since the number of tomatoes on her plants are twelve consecutive whole numbers, then her plants must *each* have the same number of extra tomatoes more than our initial assumption. How many extra tomatoes should each plant have? 108 tomatoes spread over 12 plants gives 9 extra tomatoes each.
Therefore, the plants have 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, and 21 tomatoes, and the last one has 21 tomatoes. (We can check that there are indeed 186 tomatoes by adding $10+11+\ldots+21$.)
ANSWER: (D)

24. Since $ABCD$ is a square and has an area of 25 cm^2, then the square has a side length of 5 cm. Since $PQCD$ is a rhombus, then it is a parallelogram, so its area is equal to the product of its base and its height.
Join point P to X on AD so that PX makes a right angle with AD, and to Y on DC so that PY makes a right angle with DC.



Then the area of the shaded region is the area of rectangle $ABZX$ plus the area of triangle PXD.
Since the area of $PQCD$ is 20 cm^2 and its base has length 5 cm, then its height, PY, must have length 4 cm.
Therefore, we can now label $DX=4$, $DP=5$ (since $PQCD$ is a rhombus), $AX=1$, and $AB=5$.
So $ABZX$ is a 1 by 5 rectangle, and so has area 5 cm^2.
Triangle PXD is right-angled at D, and has $DP=5$ and $DX=4$, so by Pythagoras' Theorem, $PX=3$. Therefore, the area of triangle PXD is $\frac{1}{2}(3)(4)=6\text{ cm}^2$.
So, in total, the area of the shaded region is 11 cm^2. ANSWER: (C)

25. Since all three numbers on the main diagonal are filled in, we can immediately determine what the product of the entries in any row, column or diagonal is, namely $6 \times 12 \times 24 = 1728$.
We can immediately start to fill in the square by filling in the top-centre, left-centre and bottom-right entries, since we have two entries in each of these rows, columns or diagonals, so the remaining entry is the overall product divided by the two entries already present.
Thus, we obtain

N	$\frac{1728}{24N}$	24
$\frac{1728}{6N}$	12	
6		$\frac{1728}{12N}$

Simplifying, we get

N	$\frac{72}{N}$	24
$\frac{288}{N}$	12	
6		$\frac{144}{N}$

In a similar way, we can fill in the two remaining entries to get

N	$\frac{72}{N}$	24
$\frac{288}{N}$	12	$\frac{1}{2}N$
6	$2N$	$\frac{144}{N}$

Now we are told that each of the nine entries is a positive integer, so each of N, $2N$, $\frac{1}{2}N$, $\frac{72}{N}$, $\frac{144}{N}$, and $\frac{288}{N}$ is a positive integer.
Do we need to check each of these conditions?
Well, if N is an integer, then $2N$ is an integer, so we don't need to check this second condition.
The fact that $\frac{1}{2}N$ is an integer tells us that N has to be an even integer.
The fact that $\frac{72}{N}$ is an integer tells us that N is a factor of 72.
The fact that $\frac{144}{N}$ is an integer tells us that N is a factor of 144, but since N is already a factor of 72 and $144 = 2 \times 72$, then N being a factor of 72 tells us that N is a factor of 144.
Similarly, N being a factor of 72 tells us that N is a factor of 288, so $\frac{288}{N}$ is an integer.
In summary, we are looking for positive integers N which are even and factors of 72.
Writing out the positive factors of 72, we get 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72, of which nine are even.

ANSWER: (C)