2004 Fryer Contest (Grade 9)

Thursday, April 15, 2004

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1. Lloyd is practising his arithmetic by taking the reciprocal of a number and by adding 1 to a number. Taking the reciprocal of a number is denoted by $\xrightarrow{R}$ and adding 1 is denoted by $\xrightarrow{A}$.
Here is an example of Lloyd's work, starting with an input of 2:

$$2 \xrightarrow{R} \tfrac{1}{2} \xrightarrow{A} \tfrac{3}{2} \xrightarrow{R} \tfrac{2}{3} \xrightarrow{A} \tfrac{5}{3} \xrightarrow{R} \tfrac{3}{5}$$

(a) Using an input of 3, fill in the five blanks below:

$$3 \xrightarrow{R} \underline{\qquad} \xrightarrow{A} \underline{\qquad} \xrightarrow{R} \underline{\qquad} \xrightarrow{A} \underline{\qquad} \xrightarrow{R} \underline{\qquad}$$

(b) Using an input of x, use the same operations and fill in the five blanks below:

$$x \xrightarrow{R} \underline{\qquad} \xrightarrow{A} \underline{\qquad} \xrightarrow{R} \underline{\qquad} \xrightarrow{A} \underline{\qquad} \xrightarrow{R} \underline{\qquad}$$

(c) Using the five steps from (b), what input should you begin with to get a final result of $\frac{14}{27}$? Justify your answer.

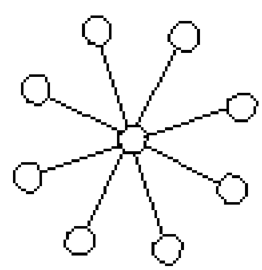
2. The Fryer Foundation is giving out four types of prizes, valued at \$5, \$25, \$125 and \$625.

(a) The Foundation gives out at least one of each type of prize. If five prizes are given out with a total value of \$905, how many of each type of prize is given out? Explain how you got your answer.

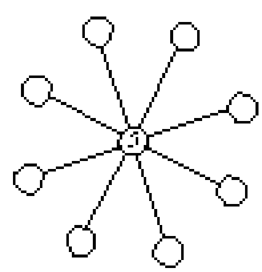
(b) If the Foundation gives out at least one of each type of prize and five prizes in total, determine the other three possible total values it can give out. Explain how you got your answer.

(c) There are two ways in which the Foundation could give away prizes totalling \$880 while making sure to give away at least one and at most six of each prize. Determine the two ways of doing this, and explain how you got your answer.

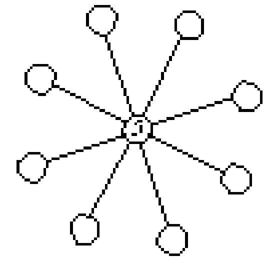
3. In "The Sun Game", two players take turns placing discs numbered 1 to 9 in the circles on the board. Each number can only be used once. The object of the game is to be the first to place a disc so that the sum of the 3 numbers along a line through the centre circle is 15.



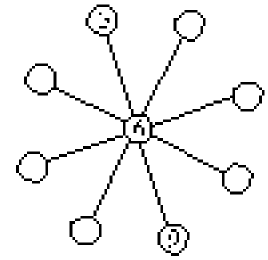
(a) If Avril places a 5 in the centre circle and then Bob places a 3, explain how Avril can win on her next turn.



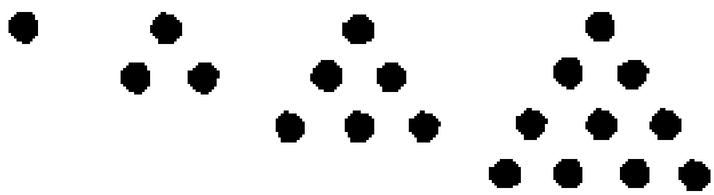
(b) If Avril starts by placing a 5 in the centre circle, show that whatever Bob does on his first turn, Avril can always win on her next turn.



(c) If the game is in the position shown and Bob goes next, show that however Bob plays, Avril can win this game.



4. Triangular numbers can be calculated by counting the dots in the following triangular shapes:



The first triangular number is 1, the second is 3, the third is 6, the fourth is 10, and the nth triangular number equals $1+2+3+\cdots+(n-1)+n$.

(a) Calculate the 10th and 24th triangular numbers.

(b) Prove that the sum of any three consecutive triangular numbers is always 1 more than three times the middle of these three triangular numbers.

(c) The 3^{rd}, 6^{th} and 8^{th} triangular numbers (6, 21 and 36) are said to be in arithmetic sequence because the second minus the first equals the third minus the second, ie. $21-6=36-21$. Also, the 8^{th}, 12^{th} and 15^{th} triangular numbers (36, 78 and 120) are in arithmetic sequence. Find three other triangular numbers, each larger than 2004, which are in arithmetic sequence.